$$\mathrm{div}(\mathbf{E}\times\mathbf{B}) = \mathbf{B}.\mathrm{rot}\mathbf{E} - \mathbf{E}.\mathrm{rot}\mathbf{B}, \tag{14.2}$$

we can write (14.1) in the form

$$\frac{\partial}{\partial t}\frac{E^2 + B^2}{8\pi} + \mathbf{J}.\mathbf{E} + \frac{c}{4\pi}\,\mathrm{div}(\mathbf{E}\times\mathbf{B}) = 0. \tag{14.3}$$

Let us now integrate this equation over an arbitrary volume V containing our electromagnetic system and use the Gauss teorem for the last term

$$\frac{\partial}{\partial t}\int_V \frac{E^2 + B^2}{8\pi}\,dV + \int_V \mathbf{J}.\mathbf{E}\,dV + \oint_S c\frac{\mathbf{E}\times\mathbf{B}}{4\pi}.d\mathbf{S} = 0, \tag{14.4}$$

where the last integral is spread over the surface S of the volume V.

Taking into account the second equation (9.14), we can write

$$\int_V \mathbf{J}.\mathbf{E}\,dV = \sum_{i=1}^{n} q_i \mathbf{v}_i.\mathbf{E}, \tag{14.5}$$

where n is the number of the charges in the system.

Putting this into (14.4) and taking into account equation (8.7), assuming there $S = -\,\mathrm{div}\mathbf{A} = 0$, as this is a rather ad hoc introduced term, we obtain

$$\frac{\partial}{\partial t}\int_V \frac{E^2 + B^2}{8\pi}\,dV + \frac{d}{dt}\sum_{i=1}^{n} e_{0i} + \frac{c}{4\pi}\oint_S (\mathbf{E}\times\mathbf{B}).d\mathbf{S} = 0. \tag{14.6}$$

If we consider the integral on the right side as time (kinetic) energy, then, having in mind the energy conservation law (2.15), we have to assume that the corresponding "particles" move with the velocity c away from the volume V and that in a unit of time the energy

$$\mathbf{I} = \frac{c}{4\pi}\,\mathbf{E}\times\mathbf{B} \tag{14.7}$$

crosses a unit surface placed at right angles to I, which is called (ELECTROMAGNETIC) ENERGY FLUX DENSITY. The quantity

$$\mathbf{S} = \frac{1}{4\pi}\,\mathbf{E}\times\mathbf{B} \tag{14.8}$$

is the density of this energy (at a snap shot) and is called the POYNTING VECTOR.

It turns out (see Chapter IV) that E and B in the last term of (14.6) are to be considered as the electric and magnetic intensities radiated by the charges of the system and thus are to be denoted by $\mathbf{E}_{rad}$ and $\mathbf{B}_{rad}$. Then E and B in the first term of (14.6) are to be considered as the radiation electric and magnetic intensities radiated by the charges of the system which still have not left the volume V and thus are also to be denoted by E_{rad} and B_{rad}. The middle term in (14.6) is the change of the time energy of the system which, according to formulas (14.5) and (8.7), is equal to the change of the potential electric energy of the system. Thus, for a given short time interval, the change of electric (or time) energy of the system is equal to the change of the radiated energy in the volume V (given by the first term in (14.6)) plus the energy radiated outside the volume V (given by the third term in (14.6)). Thus E and B in formula (14.6) do not represent the potential electric and magnetic intensities, $\mathbf{E}_{pot}$, B_{pot}, but only the radiation electric and magnetic intensities E_{rad}, B_{rad}. In Chapter IV we shall see that $E_{rad} = B_{rad}$ and $E_{rad}.B_{rad} = 0$.

Considering the potential electric and magnetic fields as physical realities, official physics brought into the theory a big mess. I repeat, the potential electric and magnetic intensities are mathematical quantities which exist only in our heads. They have neither energy density (the energy density near the charges will be infinitely big and thus incalculable!) nor momentum density. Meanwhile the radiated electric and magnetic intensities are physically existing quantities with the energy density

$$\bar{S} = (E^2 + B^2)/8\pi \qquad (14.9)$$

and momentum density **I** given by formula (14.7).

Concluding this chapter, let me say that the Maxwell-Lorentz equations are not some "physical" equations invented by somebody. They are the most trivial mathematical deductions from the Newton-Lorentz equation (which in its official form can be found in Maxwell's "Treatise" and thus it is unjustified to call it "Lorentz equation") and the equations (9.16) connecting densities and potentials, which, from their part, are the most obvious results of the definition equations (8.1) for the potentials and the definition equations (9.14) for the densities.

But neither the Newton-Lorentz equation is some "physical" equation, as it is a trivial mathematical result from the Coulomb law (axiom V), the Neumann law (axiom VIII), the form of the time energy of mass m moving with velocity **v** (axiom VI) and the energy conservation law (axiom IX). I have, however, to emphasize that I spent 3 years in Sofia of intensive mental work some 20 years ago to arrive at the deduction of the Lorentz equation from the mentioned four axioms, and my last 10 years in Graz to understand that at this deduction I had to take dA/dt in the form(7.9) and not without the term **v**div**A**, as I did in Sofia, and to write it thus in the Newton-Lorentz form. Nicolaev's experiments, however, impelled me to introduce some changes in this term (see Sect. 24).

Thus, according to me, in classical physics there are only four discoveries:

1) Coulomb's law in electromagnetism and Newton's law in gravitation.

2) Neumann's law (as a matter if fact, the coronation of Neumann's law as a fundamental physical axiom was done by me).

3) The form of the time energy of a particle.

4) The energy conservation law.

As my own physical discovery, I consider the revelation of the Marinov-aether character of light propagation. In my CLASSICAL PHYSICS[5] the Marinov-aether character of light propagation is introduced in the theory as an axiom (the tenth axiom). I did not follow this way in the present book, as the volume of Sect. 2 had to be substantially increased, meanwhile I wish to explain with this book what electromagnetism is in the most laconic way.

As another physical discovery is to be considered the introduction, rather *ad hoc*, of the scalar magnetic intensity in its Whittaker's and Nicolaev's forms (see Sect. 24), noting, however, that the form of the scalar magnetic intensity is still not established definitely. The "discovery" of the motional-transformer induction and the "invention" of the perpetua mobilia MAMIN COLIU, VENETIN COLIU and SIBEREAN COLIU (see Chapter VI) are simple logical results to which all logically thinking children have to come alone when analyzing the Newton-Lorentz equation. Thus, according to me, discovery is the creation of an axiomatical assertion (which is right!). The mathematical deductions from the axiomatical assertions cannot be discoveries.

I do not consider the coronation of the potentials as the primary physical quantities and the decoronation of the intensities as an achievemnt of some value, as those are obvious things and every logically thinking child has to come alone to these conclusions. Indeed, if **A** is given, then every ordinary child is able the calculate quickly E_{tr}, **B** and S, but if E_{tr}, **B** and S are given neither the most extraordinary professor is able to calculate **A**.

Neither the establishment of space and time as absolute categories nor the rejection of the principles of relativity and equivalence can be considered as achievements of some value, as every normally thinking child accepts these assertions as true and not the oposite.

III. LOW-ACCELERATION ELECTROMAGNETISM

15. INTRODUCTION

Further I shall no more pay attention to gravimagretism and only some "neuralgic" aspects of electromagnetism will be treated.

In Chapter III the acceleration of the electric charges of the system considered will be supposed low and thus their radiation will be neglected (it will be shown in Chapter IV that the energy radiated by the electric charges is proportional to their accelerations).

The electromagetic equations obtained in Chapter II are for a system of single particles. But the electromagnetic systems with which we experiment only rarely consist of single particles. The predominant part of the material systems are MEDIA which are built in a very complicated manner of single charged and uncharged particles. We shall disregard the way in which the media are built and we shall accept very simple models elaborated by humanity after centuries of experimental work and observations. It turns out that by accepting these genuine models of the media, we can calculate a large quantity of the electromagnetic phenomena by the help of the simple equations deduced in Chapter II for a system of single particles. This simple approach to the problems of electromagnetism is called PHENOMENOLOGICAL APPROACH.

I shall work in this book with the most simple media: current conducting wires, condensers filled by air (vacuum) or by dielectrics and coils filled by air or by magnetics, appealing to the most general and elementary knowledges of the reader, elaborated in the secondary schools or by reading some popular booklets.

16. RESISTANCE

The ELECTRIC CURRENT I which flows in a metal wire (which will be called also CONDUCTOR) is the quantity of electric charge dq which crosses its cross-section for the time dt

$$I = dq/dt. \tag{16.1}$$

The electric tension dU along a length dr of the conductor will be given by formula (13.7), where E will be the acting electric intensity which I call also DRIVING ELECTRIC INTENSITY. Consequently the tension U along the whole or a part of the conductor will be called DRIVING ELECTRIC TENSION.

It was experimentally established (by Ohm in 1826) that the current flowing in a conductor is proportional to the electric tension between its end points

$$I = GU, \tag{16.2}$$

where the coefficient G which depends on the material substance of the conductor and on its geometry is called CONDUCTANCE. Equation (16.2) is called OHM'S LAW.

The conductance of a wire with a unit length and unit cross-section is called CONDUCTIVITY and is denoted by γ. Thus the conductance of a wire with length L and cross-section S will be

$$G = \gamma S/L. \tag{16.3}$$

RESISTANCE R, which is much more used in practice, is the quantity inverse to conductance

$$R = 1/G = L/\gamma S = \rho L/S, \tag{16.4}$$

where ρ is called RESISTIVITY and this is the resistance of wire with unit length and unit cross-section. Thus we can write

$$I = U/R. \tag{16.5}$$

If the resistance of a wire is zero, it is called SUPER-CONDUCTOR.

Let us suppose that dq charges have been transferred along a conductor for a time dt, the tension between whose end points is $U = \Delta\Phi$, where $\Delta\Phi$ is the difference between the electric potentials at the end points. According to the first formula (8.2), in which we have to write U_e, dq and $\Delta\Phi$ instead of U, q and Φ, the electric energy of the system will change with

$$dU_e = dq\Delta\Phi = dqU = IUdt, \tag{16.6}$$

where equation (16.1) was taken into account.

The change of the energy in a time unit

$$P = dU_e/dt \tag{16.7}$$

is called POWER, and from (16.6) and (16.7) we obtain

$$P = IU = RI^2 = U^2/R. \tag{16.8}$$

This power is liberated as heat in the conductor and is lost by the source supplying the driving tension. HEAT is a physical phenomenon outside the domain of electromagnetism and for this reason Ohm's law cannot be obtained from my axiomatics. In "pure" electromagnetism, which is to be thoroughly explained by logical deductions from the axiomatics, the conductors must be super-conductors.

Until the present time it is not clear how electric current propagates along metal wires. The phenomenological model proposed by me[6] is the following:

The so-called valence electrons, which are the current conducting electrons, are loosely connected with the ions of the metal lattice, jumping continuously from one atom to another and forming a kind of "electron gas" throughout the solid ions' lattice. If there is no electric tension applied to the wire, the motion of the valence electrons is chaotic and their average velocity is zero. When an electric tension is applied to the wire (imagine, for simplicity, that an electric pulse is applied to the left end of the wire by supplying a surplus of electrons), the chaotically moving electrons from the left end, where the concentration exeeds the concentration of the valence electrons, begin to move with a preferred average velocity to the right, where the electron concentration is less. The average "DRIFT

VELOCITY" of the electrons, v_{dr}, is of the order of mm/sec. This velocity can be easily calculated if assuming that all valence electrons in the wire are current conducting electrons. However the velocity, v_{en}, with which the "electrons' concentration" propagates through the wire, and which I call the ENERGY VELOCITY, is of the order of c, as can be established by measuring the velocity with which the current pulse propagates. Thus, after a second the exceeding electrons which were supplied to the left wire's end will be transferred to 1 mm, but the electrons' concentration will be exceeding at a distance of 300,000 km. If the wire is not closed, the electrons' concentration will be reflected from the right end and returning back will be reflected from the left end, and so on, until the surplus electrons will be distributed uniformly troughout the wire and its surface will become equipotential.

As the electrons are absolutely identical and indistingushable one from another, we must conclude that in a second the exceeding electrons were transferred at a distance of 300,000 km. (Indeed, if 100 electrons in file move on 1 cm each in a second or the first electron moves on 100 cm, while the other 99 remain at rest, the physical result is the same.)

If there is a consumer at the right end of the wire and the supply of surplus electrons at the left end is continuous, the electric energy from the supplier to the consumer will proceed along the wire with the velocity $v_{en} = c$.

It must be clear that the velocity of the single electrons is neither the drift velocity, v_{dr}, nor the energy velocity, v_{en}. Every electron moves chaotically. It is possible that some of the supplied surplus electrons may cover the whole wire with a velocity c and be always in the "electrons' surplus concentration". The probability for such a case is v_{dr}/v_{en}. Even in a wire without electric tension there is a possibility that some electron will cross it from one end to the other with a velocity c, however the probability for such a case is zero. Although the electric energy transferred along a wire is something material and can be measured in energy units transferred in a time unit along a length unit, official physics speaks about a foggy "propagation of interactoon", being unable to explain what a physical quantity "interaction" is and with which measuring instruments and in which measuring units is to be measured. For certain official physicists the "interaction" propagates through the metal, for other it surrounds the conductor similarly to the aura which surrounds the human body according to the assertions of the Indian yogas.

My friends Milnes[(10)] and Pappas[(11)] have done experiments for measuring the velocity of propagation of current pulses along copper wires and have established that it is much higher than c, at least 10 or even 100 times higher than c.

It turns out that only the directed motion of the electrons liberates heat but the chaotic motion does not. This result makes the hypothesis about the "electron gas" shaky. Thus after so many years of experimentation with currents in metal wires one can make the conclusion: we still do not know the mechanism of propagation of the current.

17. CAPACITANCE

It is obvious that the potential difference (tension) between a charged conductor and other uncharged conductors in its neighbourhood (the latter usually are connected to earth) will be proportinal to the electric charge q on the conductor

$$U = (1/C)q, \tag{17.1}$$

where the coefficient 1/C depends on the geometry of the whole system and C is called CAPACITANCE. The number C shows the quantity of electric charge with which the conductor is to be charged to increase its potential with unity respectively to the uncharged conductors. A material system which has capacitance is called CONDENSER (one can use also the word CAPACITOR).

Let us have a condenser consisting of two parallel plates of surface S, the distance between which is d. One can use equation (13.8) and the second equation (13.2) to find its capacitance. The volume of integration V will be chosen so that it contains one of the plates, the charge density on which is Q. Designating the surface of the volume V by S', we shall have

$$\oint_{S'} E.dS = 4\pi \int_V Q dV = 4\pi q, \tag{17.2}$$

where q is the whole charge on the plate (the charge on the other plate is -q if the latter is not earthed). If d is small with respect to $\sqrt{S}$, we can assume that the electric intensity is different from zero only between the plates, being there constant and perpendicular to the plates. Thus we shall have

$$ES = 4\pi q. \tag{17.3}$$

As E = U/d, we obtain from here

$$q = (S/4\pi d)U. \tag{17.4}$$

Comparing this with (17.1), we obtain for the capacitance of the parallel plate condenser

$$C = S/4\pi d. \tag{17.5}$$

We see from equation (17.4), if denoting the surface charge density by $\Sigma = q/S$, that the electric intensity between two nearly placed parallel plates charged homogeneously with surface charge density Σ is

$$E = 4\pi\Sigma. \tag{17.6}$$

Let us find now the capacitance of a cylindrical condenser with coaxial plates with radii R_i and R_e of the internal and external plates and length L, supposing $R_e - R_i \ll L$.

We use again formula (13.8) and choose the volume of integration V to contain only the internal cylindrical plate. Assuming again that E is different from zero only in the space between the plates where it is constant and perpendicular to the condenser's axis, we shall obtain from (13.8), if choosing the integration surface crossing the space between the plates to be cylindrical with a radius r,

$$E(2\pi rL) = 4\pi q. \qquad (17.7)$$

Thus the tension between the plates will be

$$U = \int_{R_i}^{R_e} E.dr = (2q/L)\int_{R_i}^{R_e} dr/r = 2q\ln(R_e/R_i)/L. \qquad (17.8)$$

Comparing this with (17.1), we obtain for the capacitance of the cylindrical condenser

$$C = L/2\ln(R_e/R_i). \qquad (17.9)$$

Denoting the surface charge density on the internal cylindrical plate by $\Sigma = q/2\pi R_i L$, we see from equation (17.7) that the electric intensity between two nearly placed coaxial cylindrical plates charged homogeneously with surface charge density Σ, at a distance r from the cylindrical axis, is

$$E = 4\pi\Sigma R_i/r. \qquad (17.10)$$

From here, at $r = R_i$, we obtain formula (17.6)

18. INDUCTANCE

18.1. INDUCTANCE OF A LOOP.

Let us have a circuit in which current I flows. This current will generate the magnetic potential A(r) at a reference point with radius vector **r**. Let us take the line integral of **A** along a certain closed loop L. According to Stokes theorem, taking into account the second formula (8.6), we shall have

$$\oint_L \mathbf{A}.d\mathbf{r} = \int_S \mathrm{rot}\mathbf{A}.d\mathbf{S} = \int_S \mathbf{B}.d\mathbf{S} = \Phi, \qquad (18.1)$$

where S is an arbitrary surface spanned on the closed line L and Φ is called MAGNETIC FLUX (electric potential and magnetic flux are designated by the same symbol and be attentive to not confound them!) crossing the surface S.

If denoting by $\mathbf{A}_0$ the magnetic potential generated by a <u>unit current</u> flowing in the circuit, and if taking the line L to be the circuit itself, we shall have

$$\Phi = I\oint_L \mathbf{A}_0.d\mathbf{r} = LI, \qquad (18.2)$$

where

$$L = \oint_L \mathbf{A}_0.d\mathbf{r} = \int_S \mathbf{B}_0.d\mathbf{S} \qquad (18.3)$$

is called INDUCTANCE of the circuit and $\mathbf{B}_0$ is the magnetic intensity generated by a unit current flowing in the circuit on the arbitrary surface S spanned on the circuit. Thus L is the magnetic flux generated by a unit current flowing in the circuit through any surface S spanned on the circuit.

18.2. INDUCTANCE OF A CIRCULAR LOOP.

Let us calculate the inductance of the most simple circular circuit (fig.2).

We take the reference frame with origin at the center of the loop and we shall calculate first the magnetic potential generated by an arbitrary current element at an internal (in the loop) and at an external (outside the loop) reference point, both lying on the positive x-axis. Let us denote the distance from the frame's origin to both reference points by ρ_{int} and ρ_{ext}, and from the loop's element by r_{int} and r_{ext}. The radius of the circular loop is denoted by R and the angle between the x-axis and the radius vector to the loop's element (which, for definiteness, let us consider in the first quadrant) by ϕ. The flow of the current will be taken in the positive direction (i.e., counter-clockwise).

If dq is the quantity of electric charge which for a time dt is transferred through the cross-section of the wire, we can write $dq\mathbf{v} = dq\,d\mathbf{r}/dt = Id\mathbf{r}$, where $I = dq/dt$ is the flowing current, $d\mathbf{r}$ is the line element of the loop taken along the current, and the expression $Id\mathbf{r}$ is called CURRENT ELEMENT. Resolving the vector of the current element into a horizontal and vertical components, we see that the actions of the horizontal components of two symmetric current elements in the first and fourth quadrants will annihilate one another, so that only the action of the vertical component will remain. Thus we concude that the magnetic potential at the internal and external reference points originated by both symmetric current elements in the first and fourth quadrants will be parallel to the y-axis. For the absolute value, according to the definition formula for $\mathbf{A}$ (8.1), we obtain

$$dA = 2\frac{I\,dr\cos\phi}{cr} = \frac{2IR\cos\phi\,d\phi}{c(\rho^2 - 2\rho R\cos\ \ + R^2)^{1/2}}, \qquad (18.4)$$

where by r and ρ either the internal or external distances are denoted, and we put $dr = Rd\phi$.

To obtain the magnetic potential originated by the current in the whole loop, we have to integrate formula (18.4) for ϕ changing from 0 to π, thus obtaining

$$A = \int dA = \frac{2IR}{c}\int_0^\pi \frac{\cos\phi\,d\phi}{(\rho^2 - 2\rho R\cos\phi + R^2)^{1/2}} = \begin{cases} \dfrac{\pi I}{c}\dfrac{\rho}{(R^2 - \rho^2)^{1/2}} & (\text{for } \rho < R), \\[2ex] \dfrac{\pi I}{c}\dfrac{R^2}{(\rho^2 - R^2)^{1/2}} & (\text{for } \rho > R). \end{cases} \qquad (18.5)$$

The value of the elliptical integral in (18.5) can be found in a standard table of integrals. This formula shows that the magnetic potential increases rapidly from 0 at the center of the loop to infinity at the loop, and then it decreases slowly to 0 at infinity.

As the magnetic potential of a circular loop has rotational symmetry, the magnetic intensity produced by it can be calculated immediately, using the expression for rotation in cylindrical coordinates, taking $\mathbf{A} = (A_\rho, A_\phi, A_z) = (0, A, 0)$, where for

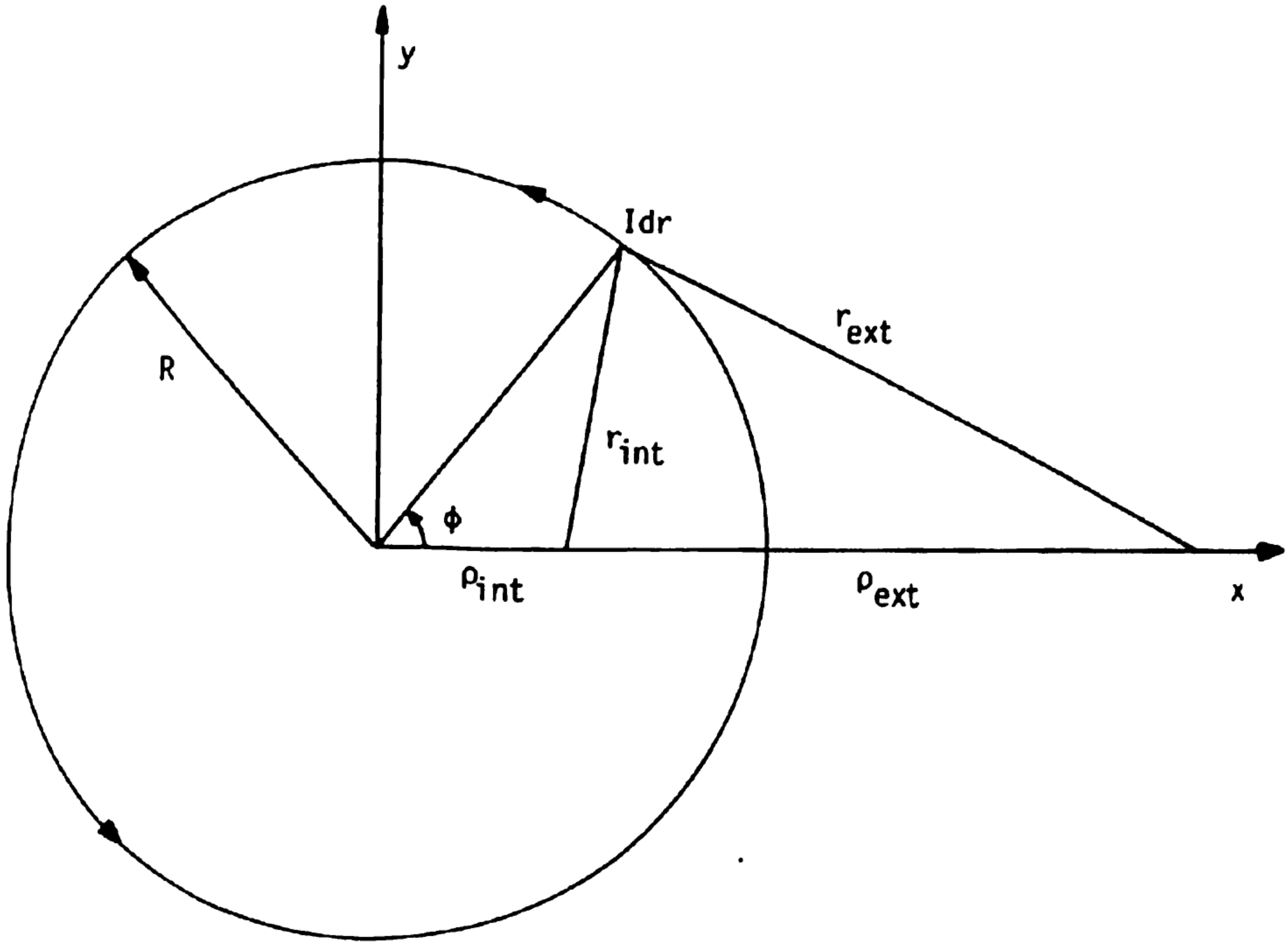

Fig. 2. Circular loop in which current flows.

A the expressions (18.5) are to be taken,

$$\mathbf{B} = \mathrm{rot}\mathbf{A} = \frac{1}{\rho}\frac{\partial(\rho A)}{\partial\rho}\hat{z} = \begin{cases} \dfrac{\pi I}{c}\dfrac{2R^2 - \rho^2}{(R^2 - \rho^2)^{3/2}}\hat{z} & (\text{for } \rho < R), \\ -\dfrac{\pi I}{c}\dfrac{R^2}{(\rho^2 - R^2)^{3/2}}\hat{z} & (\text{for } \rho > R). \end{cases} \quad (18.6)$$

This formula shows that the magnetic intensity increases from $(2\pi I/cR)\hat{z}$ at the center of the loop to $\infty\hat{z}$ at the loop inside and then decreases from $-\infty\hat{z}$ at the loop outside to 0 at infinity.

Let us calculate the inductance of the circular circuit according to the second formula (18.3) for $\rho < R$

$$L = \int_S \mathbf{B}_0 . d\mathbf{S} = \frac{\pi}{c}\int_0^R \frac{2R^2 - \rho^2}{(R^2 - \rho^2)^{3/2}} 2\pi\rho\, d\rho = \frac{2\pi^2}{c}R + \frac{2\pi^2}{c}\frac{R^2}{(R^2 - \rho^2)^{1/2}}\Bigg|_0^R = \infty. \quad (18.7)$$

We see that by substituting the limit "R" in the solution on the right side, we obtain infinity. Thus the inductance of a circular infinitely thin loop is infinitely large.

If the radius of the circular wire is r, we have to divide the integral (18.7) into two integrals: one in the limits from 0 to $R - r$, in which the magnetic intensity in the circle of radius $R - r$ is generated by the whole current (in our case $I = 1$), and one in the limits from $R - r$ to R, in which the current is a function of the integration variable. In our case we have to take $I = (R-\rho)/r$, if the current is distributed homogeneously across the wire's cross-section. As, however, one needs to know the inductance for alternating currents, we have to take into account the SKIN EFFECT, according to which there will be more current near the cylindrical surface of the conductor and less along its axis, so that the calculation becomes more complicated and the inductance of a loop becomes dependent on the current frequency.

Thus, for homogeneously distributed current, the integral (18.7) must be separated into the following two integrals

$$L = \frac{\pi^2}{c}\int_0^{R-r} \frac{2R^2 - \rho^2}{(R^2 - \rho^2)^{3/2}}\, d(\rho^2) + \frac{\pi^2}{c}\int_{R-r}^{R} \frac{(R-\rho)(2R^2 - \rho^2)}{r(R^2 - \rho^2)^{3/2}}\, d(\rho^2). \quad (18.8)$$

The calculation of the second integral is pretty complicated but in a good approximation (good enough for any practical use) we can solve it as follows: Let us multiply and divide the second integrand by $R+\rho$ and let us put $\rho = R$ everywhere in the second integrand besides the expressions $R^2 - \rho^2$.

The values of the two integrals in (18.8) will be

$$L = \frac{2\pi^2(R - r)^2}{c(2rR - r^2)^{1/2}} + \frac{\pi^2 R(2R - r)}{c(2rR - r^2)^{1/2}} \simeq \frac{4\pi^2 R^2}{c(2rR)^{1/2}} = \frac{2\sqrt{2}\pi^2 R^{3/2}}{c\sqrt{r}}, \quad (18.9)$$

where the result on the right is obtained by neglecting r with respect to R.

Thus the first integral in (18.8) gives only the half of the right value.

Scott[12] also tried to find the inductance of a circular wire and after horrible calculations, where the physical substance of the problem was completely lost, obtained the following result

$$L_{Scott} = (4\pi/c)R\{\ln(8R/r) - 7/4\}. \quad (18.10)$$

Scott's formula is definitely wrong, as the truncated first integral (18.8), which I shall denote by L_{trunc} and which gives a value definitely lower than the true inductance L_{true}, is always larger than L_{Scott}. Here are the relations L_{trunc}/L_{Scott} for R/r = 10; 100; 1000: L_{trunc}/L_{Scott} = 1.11; 2.21; 4.84. The relations of the true enough inductance L given by the value on the right of (18.9) to Scott's value for the same ratios R/r are: L/L_{Scott} = 2.74; 4.51; 9.71.

There are also two aesthetical reasons showing that Scott's formula is wrong: 1) His theretical demonstration is too complicated and MARINOV'S RAZOR says: *Ogni teoria complicata è sbagliata*. 2) The number 7/4 indicates that something is rotten in the formula: the Divinity cannot put this number in a formula describing such a symmetric effect.

King[13] gives in *Handbuch der Physik*, the most authoritative source of physics knowledge, the following formula for the inductance of a circular loop

$$L_{king} = (4\pi/c)\{R(R+r)\}^{1/2}\{(2/k - k)K(\pi/2,k) - (2/k)E(\pi/2,k)\}, \quad (18.11)$$

where

$$k = (1 - k'^2)^{1/2}, \qquad k' = r/(2R + r), \quad (18.12)$$

and

$$K(\pi/2,k) = \int_0^{\pi/2} \frac{d\phi}{(1 - k^2\sin^2\phi)^{1/2}}, \qquad E(\pi/2,k) = \int_0^{\pi/2} (1 - k^2\sin^2\phi)^{1/2}d\phi \qquad (18.13)$$

are the complete elliptic integrals of first and second kinds which are tabulated as functions of k.

When R/r is sufficiently large, k' is small and the elliptic integrals may be expounded in powers of K'. For the leading terms King has obtained

$$L_{king} = (4\pi/c)R\{\ln(8R/r) - 2\}. \qquad (18.14)$$

King's formula is very near to Scott's formula, and this is an indication that both authors have calculated well. Why then are their formulas wrong? - According to me, the explanation for the substantial difference between my formula (18.9), on one side, and formulas (18.10) and (18.14) of Scott and King, on the other side, is that they have done the calculations proceeding from the first formula (18.3) (as a matter of fact, from (18.16)), while I did the calculation proceeding from the second formula (18.3). My way is mathematically simple and straighforward, the ways of Scott and King are horribly complicated, as they lead to elliptical integrals.

Nevertheless, as the left and right formulas (18.3) are mathematically indentical, one has to obtain identical results. I leave to the mathematicians the honour to find why the calculations of Scott and King have led to a wrong result.

18.3. NEUMANN'S FORMULA.

Returning to formula (18.3) and taking into account that

$$\mathbf{A} = \oint_L I d\mathbf{r}/cr, \qquad \mathbf{A}_0 = \oint_L d\mathbf{r}/cr, \qquad (18.15)$$

we can write the left side of formula (18.3) in the form

$$L = \oint_L \oint_L d\mathbf{r}.d\mathbf{r}'/cr. \qquad (18.16)$$

Let us have now two circuits L_1 and L_2. Let us take the line integral of the magnetic potential $\mathbf{A}_1$ generated by the current I_1 in the first circuit along the contour L_2 of the second circuit. Using again Stokes theorem, as in formula (18.1), we shall have

$$\oint_{L_2} \mathbf{A}_1.d\mathbf{r}_2 = \int_{S_2} \text{rot}\mathbf{A}_1.d\mathbf{S}_2 = \int_{S_2} \mathbf{B}_{12}.d\mathbf{S}_2 = \Phi_{12}, \qquad (18.17)$$

where S_2 is an arbitrary surface spanned on the closed line L_2 and Φ_{12} is the magnetic flux generated by the current in the loop L_1 which crosses the surface of the loop L_2. If $\mathbf{A}_1$ is generated by a unit current and if taking into account formula (18.15), we can write for the MUTUAL INDUCTANCE of L_2 due to the unit current in L_1

$$L_{12} = \oint_{L_2} \mathbf{A}_{01}.d\mathbf{r}_2 = \oint_{L_1}\oint_{L_2} d\mathbf{r}_1.d\mathbf{r}_2/cr_{12}. \qquad (18.18)$$

This is called the FORMULA OF NEUMANN and obviously $L_{12} = L_{21}$.

Now the inductance (18.16) can be called SELF-INDUCTANCE and denoted by L_{11}.

If we have N circular loops with the same radius overlapping one another and if the common radius of their filaments is much less than the loop's radius, we can make the following conclusion: The self-inductance of every loop will be L (see (18.9)) and the mutual inductance of every loop caused by the other N-1 loops will be (N-1)L. Thus the inductance of all N loops will be N^2L.

If the distances between the loops are considerable and their positions one with respect to another arbitrary, every single mutual inductance will be less than L, and thus the inductance of the whole system will be less than N^2L.

Let me note that if the currents I_1 and I_2 are flowing, respectively, in the coils L_1 and L_2, the mutual inductance of whom is L_{12}, then the mutual magnetic energy of the currents in these two coils will be (see (2.14) and (18.18))

$$W_{12} = - \oint_{L_1}\oint_{L_2} q_1\mathbf{v}_1.q_2\mathbf{v}_2/c^2r_{12} = - \oint_{L_1}\oint_{L_2} I_1d\mathbf{r}_1.I_2d\mathbf{r}_2/c^2r_{12} = - I_1I_2L_{12}, \qquad (18.19)$$

where the relations $I_1d\mathbf{r}_1 = q_1\mathbf{v}_1$, $I_2d\mathbf{r}_2 = q_2\mathbf{v}_2$ have been taken into account.

As a matter of fact, I called equation (2.14) Neumann's law when proceeding from formula (18.19).

For the magnetic energy of the current elements in a single coil with self-inductance L we shall have

$$W = - (1/2)LI^2 \qquad (18.20)$$

and it is a negative quantity, meanwhile in any official text-book on electromagnetism this energy is taken wrongly as a positive quantity.

It is easy to see that on the right side of (18.20) the coefficient 1/2 is to be taken, as at the integration in (18.16) we take once the product of $d\mathbf{r}_i$ with $d\mathbf{r}_j'$ and once the product of $d\mathbf{r}_j$ with $d\mathbf{r}_i'$, so that we shall obtain twice their magnetic energy. Of course, we can write (18.20) without the factor 1/2 but then this factor is to be put in formula (18.16).

I have, however, to emphasize that the calculation of the self-inductance according to formula (18.16) inevitably leads to improper integrals, as the distance r_{ii} between the element $d\mathbf{r}_i$ at the one integration along L and the element $d\mathbf{r}_i' = d\mathbf{r}_i$ at the other integration along the same contour L is zero. Perhaps here is to be searched for the wrong calculations of Scott and King.

18.4. INDUCTANCE OF AN INFINITELY LONG SOLENOID.

Let us consider N circular loops of radius R with a common axis and having the same distance one from another, in which current I flows. We can assume, for mathematical rigorosity, that the N circular loops are independent and any has its own source of electric tension, but, of course, we shall have in mind that all loops are connected, building thus a COIL, and that there is only one source of electric tension. Such a cylindrical coil is called also SOLENOID. If the length of the solenoid is l, there will be n = N/l TURNS (of WINDINGS) on a unit of its length. When l tends to infinity, the solenoid is called INFINITE.

The magnetic potential in the plane of any circular loop generated by its own current is given by formula (18.5). The magnetic potential generated in a plane whose distance from the loop's plane is z will be

$$A = \frac{2IR}{c}\int_0^{\pi}\frac{\cos\phi\, d\phi}{(\rho^2 - 2\rho R\cos\phi + R^2 + z^2)^{1/2}}. \tag{18.21}$$

The magnetic potential generated by all windings of an infinite solenoid at a point with cylindrical coordinates ρ, ϕ, z will be

$$A = \sum_{i=1}^{N=\infty} A_i = \frac{2IR}{c}\int_0^{\infty} n dz \int_0^{\pi}\frac{\cos\phi\, d\phi}{(\rho^2 - 2\rho R\cos\phi + R^2 + z^2)^{1/2}}. \tag{18.22}$$

This integral can be evaluated by dividing it in two parts, from 0 to $\pi/2$ and from $\pi/2$ to π, writing in the second integral $\pi - \phi$ for ϕ and interchanging its limits. Denoting then $a_1 = \rho^2 - 2\rho R\cos\phi + R^2$, and $a_2 = \rho^2 + 2\rho R\cos\phi + R^2$, we shall have

$$A = \frac{2nIR}{c}\int_0^{\infty}dz\int_0^{\pi/2}\cos\phi\, d\phi\{\frac{1}{(a_1^2 + z^2)^{1/2}} - \frac{1}{(a_2^2 + z^2)^{1/2}}\}. \tag{18.23}$$

Interchanging now the order of integration, we can easily take the integral on z

$$A = \frac{2nIR}{c}\int_0^{\pi/2}\cos\phi\, d\phi \ln\{\frac{a_2}{a_1}\frac{z + (z^2 + a_1^2)^{1/2}}{z + (z^2 + a_2^2)^{1/2}}\}\Big|_0^{\infty} = \frac{2nIR}{c}\int_0^{\pi/2}\cos\phi\, d\phi \ln(a_2/a_1) =$$

$$\frac{2nIR}{c}\int_0^{\pi/2}\cos\phi\, d\phi \ln\{\frac{\rho^2 + 2\rho R\cos\phi + R^2}{\rho^2 - 2\rho R\cos\phi + R^2}\}. \tag{18.24}$$

Let us denote $\alpha = 2\rho R/(\rho^2 + R^2)$ and use integration by parts, the one part being $\cos\phi d\phi$ and the other the logarithm. The integrated part vanishes and the integral, except for the factor $2nIR/c$, becomes

$$2\alpha\int_0^{\pi/2}\frac{\sin^2\phi\, d\phi}{1 - \alpha^2\cos^2\phi} = \frac{2}{\alpha}\{\phi - (1 - \alpha^2)^{1/2}\arctan\frac{\tan\phi}{(1 - \alpha^2)^{1/2}}\}\Big|_0^{\pi/2}, \tag{18.25}$$

as the reader can readily verify by differentiation.

The expression $\arctan\{\tan\phi/(1 - \alpha^2)^{1/2}\}$ approaches $\pi/2$ as $\phi \to \pi/2$. Using

$$(1 - \alpha^2)^{1/2} = (\rho^4 - 2\rho^2 R^2 + \rho^4)^{1/2}/(\rho^2 + R^2) = |\rho^2 - R^2|/(\rho^2 + R^2), \tag{18.26}$$

we obtain

$$A = \frac{2nIR}{c}\frac{\rho^2 + R^2}{\rho R}\frac{\pi}{2}(1 - \frac{|\rho^2 - R^2|}{\rho^2 + R^2}). \tag{18.27}$$

Thus

$$A = \begin{matrix} 2\pi nI\rho/c, \\ 2\pi nIR^2/c\rho, \end{matrix} \qquad B = \frac{1}{\rho}\frac{\partial(\rho A)}{\partial\rho} = \begin{matrix} 4\pi nI/c, & (\text{for } \rho < R), \\ 0, & (\text{for } \rho > R). \end{matrix} \tag{18.28}$$

The inductance of one loop of this infinite solenoid, according to <u>both</u> formulas (18.3), will have the value

$$L = 4\pi^2 nR^2/c = 4\pi nS/c = 4\pi NS/cl, \quad (18.29)$$

where $S = \pi R^2$ is the cross-section of the solenoid.

The inductance of all $N = nl$ loops of the solenoid will be

$$L = 4\pi^2 n^2 lR^2/c = 4\pi n^2 lS/c = 4\pi N^2 S/cl. \quad (18.30)$$

This formula remains valid for a final solenoid if l is big enough with respect to R. Otherwise the inductance of the solenoid will be less than (18.30).

19. RESISTORS, CAPACITORS AND INDUCTORS

Every conductor has a certain resistance, capacitance and inductance. Conductors for which only one of these qualities is predominant are called, respectively, RESISTORS, CAPACITORS (condensers) and INDUCTORS. An IDEAL RESISTOR is this one whose capacitance and inductance are (or can be accepted) zeros. An IDEAL CAPACITOR is this one whose resistance and inductance are zeros. An IDEAL INDUCTOR is this one whose resistance and capacitance are zeros.

In Sect. 16 the energetic aspects of the resistors have been already considered. Let us now consider the energetic aspects of capacitors and inductors.

To charge a condenser having capacitance C with total charge q_0, we have to spent the following energy (see the first formula (8.2) in which we have to exchange the potential difference $\Delta\Phi$ by the tension U)

$$U_e = \int_0^{q_0} U dq = \int_0^{q_0} (q/C) dq = q_0^2/2C = CU_0^2/2, \quad (19.1)$$

where U and q are the variable tension and electric charge of the condenser during the charging and U_0 is the tension of the charged condenser. This energy will be invested as MECHANICAL ENERGY ("mechanical energy" is another name of kinetic energy) because always when we add a new portion of charge dq the repulsion from the side of the charges on the condenser q becomes greater and gretaer. The electric energy U_e stored in the condenser can then be liberated when discharging it.

Usually a condenser is charged by a SOURCE OF ELECTRIC TENSION. The sources of electric tension can be chemical (a CELL, called also a BATTERY), thermal (thermocouple), mechanical (friction of two solid bodies), piezoelectric (appearing at an increased pressure on a solid body), induced (see Sect. 21). Every source of electric tension has its own resistance, called internal resistance and denoted by R_i. If $R_i = 0$, the source is called IDEAL.

The tension produced by a source of electric tension is called usually DRIVING (ELECTRIC) TENSION and is denoted by U_{dr}. For U_{dr} official physics uses the very bad term ELECTROMOTIVE FORCE. Also the very bad term VOLTAGE is used for electric tension.

A charged condenser is also a source of electric tension. If we connect its plates by a conductor with zero resistance, it will discharge momentarily with an infinitely large current.

Let now discharge a condenser with capacitance C through a resistor with resistance R. The sum of the tensions on the condenser and on the resistor must be zero and thus we can write

$$RI + q/C = 0 \qquad \text{or} \qquad Rdq/dt = - q/C, \tag{19.2}$$

where q is the charge on the condenser at the moment t. The differential equation (19.2) can be solved directly and its integral is

$$\int_{q_0}^{q} dq/q = - (1/RC) \int_0^t dt. \tag{19.3}$$

Taking the integral, we obtain

$$\ln(q/q_0) = - t/RC \qquad \text{or} \qquad q = q_0 e^{-t/RC}, \tag{19.4}$$

and we have further

$$I = (q_0/RC)e^{-t/RC} = (U_0/R)e^{-t/RC} = I_0 e^{-t/RC}, \qquad U = U_0 e^{-t/RC}. \tag{19.5}$$

The value RC is now seen to be the time it takes the charge, current and potential to drop to 1/e = 0.368 of its initial value and is called the TIME CONSTANT of the circuit containing the capacitance C and the resistance R.

Now if we charge up a condenser with a cell of driving tension U_{dr} and wires of total resistance R (including the eventual internal resistance R_i of the cell), the driving tension must be equal to the sum of the tensions on the resistor and on the condenser

$$U_{dr} = RI + q/C \qquad \text{or} \qquad CU_{dr} = RCdq/dt + q. \tag{19.6}$$

To solve this differential equation in the form of the indefinite integral as above, let us define the charge $Q = CU_{dr} - q$ as the difference between the final charge CU_{dr} on the condenser and its value q at any time t. Then $q = CU_{dr} - Q$ and $dq/dt = - dQ/dt$, so that equation (19.6) reads

$$CU_{dr} = - RCdQ/dt + CU_{dr} - Q, \tag{19.7}$$

or

$$dQ/Q = - (1/RC)dt. \tag{19.8}$$

Thus we obtain as above

$$Q = Q_0 e^{-t/RC}, \tag{19.9}$$

and as for q = 0 there is $Q_0 = CU_{dr}$, we have

$$CU_{dr} - q = CU_{dr} e^{-t/RC}, \tag{19.10}$$

which rearranges to

$$q = CU_{dr}(1 - e^{-t/RC}), \tag{19.11}$$

from which we derive

$$I = U_{dr} e^{-t/RC}/R, \qquad U = U_{dr}(1 - e^{-t/RC}). \tag{19.12}$$

Let us consider now an ideal inductor with inductance L.

If the current in the inductor changes, an electric tension will appear in the inductor directed oppositely to the driving tension producing the current. The value of this electric tension can be found proceeding from the Newton-Lorentz equation (8.5). Putting in this equation $\Phi = 0$, $v = 0$, as the inductor is not charged electrically and is at rest, we shall find for the global electric intensity which in this case I shall call INDUCED ELECTRIC INTENSITY

$$\mathbf{E}_{ind} = - \partial\mathbf{A}/c\partial t, \qquad (19.13)$$

where **A** is the magnetic potential along the inductor.

For the INDUCED ELECTRIC TENSION which will appear along the whole length of the inductor L (do not confound the length of the inductor with its inductance) we shall have (see (18.2))

$$U_{ind} = \oint_L \mathbf{E}_{ind}.d\mathbf{r} = - (\partial/c\partial t)\oint_L \mathbf{A}.d\mathbf{r} = - (\partial/c\partial t)\int_S \mathbf{B}.d\mathbf{S} = - \partial\Phi/c\partial t = - L\partial I/c\partial t, \qquad (19.14)$$

where **B** is the magnetic intensity through the surface S spanned over the contour L of the inductor (or the sum of the surfaces spanned on its single windings), Φ is the <u>common</u> magnetic flux and I is the current flowing in the inductor. Equation (19.14) is called FARADAY'S LAW, although it is the <u>most trivial</u> result from the Newton-Lorentz equation.

Equation (19.14) shows that only when the magnetic potential along the inductor's wires changes in time, an induced electric intensity and thus also induced electric tension do appear. And the magnetic potential changes in time only when the current changes in time.

I repeat here the statement presented in many of my articles: Electromagnetism can (and has to) be explained operating only with the potentials. One introduces the notion "intensities" (and "fluxes") only for mathematical or mnemonic conveniences. So, for example, working with the intensity and not with the potentials, I "calculated" in Sect. 18 the inductance of a circular loop much more easily than it can be done if working with the potential. On the other hand, however, the calculation with the intensities may lead to wrong results (see Sect. 22), as the intensities are <u>derivatives</u> of the potentials and contain <u>less</u> mathematical information.

Let us now make a circuit of an ideal inductor with inductance L, a resistor of resistance R and a cell with driving tension U_{dr}. The driving tension plus the induced tension must be equal to the tension on the resistor, called also OHMIC (ELECTRIC) TENSION,

$$U_{dr} + U_{ind} = U \qquad \text{or} \qquad U_{dr} = RI + LdI/cdt. \qquad (19.15)$$

Let us multiply this equation by the charge $dq = Idt$ which has passed for a time dt along the circuit, i.e., from the positive electrode of the source to its negative source, and integrate then the equation for the time from 0 to t

$$\int_0^t U_{dr} I dt = \int_0^t RI^2 dt + \int_0^{I_0} LIdI/c, \qquad (19.16)$$

where $I = 0$ is the current at the initial zero moment and $I_0 = U_{dr}/R$ is the current when $dI/dt = 0$.

The integral on the left gives the energy lost by the source, the first integral on the right gives the energy liberated as heat in the resistor and the second integral on the right gives the magnetic energy

$$W = - LI_0^2/2c \tag{19.17}$$

taken with an opposite sign, as according to equation (2.15) the electromagnetic energy of a system is equal to the difference of its electric and magnetic energies. The magnetic energy (19.17) is stored in the inductor which can be then liberated when shortcircuiting the driving tension.

At such a short-circuiting of the external driving tension U_{dr}, the driving tension in the circuit will be the induced tension and it must be equal to the ohmic tension

$$U_{ind} = U \qquad \text{or} \qquad - LdI/cdt = RI. \tag{19.18}$$

This is a differential equation of the form of the equation (19.3) and the solution, by analogy with the solution (19.4), will be

$$I = I_0 e^{-cRt/L}, \tag{19.19}$$

where $t = 0$ now refers to the time of the short-circuiting of the source.

Let us find the amount of heat liberated in the resistor. From the equation (19.18), after the multiplication by Idt and integration for the time from $t = 0$ to $t = \infty$, we obtain

$$\int_0^\infty RI^2 dt = - L \int_{I_0}^0 IdI/c = LI_0^2/2c, \tag{19.20}$$

which is just the extra amount of energy originally provided by the cell and "pumped" in the inductor. Now, at the short-circuiting of the external driving tension, this energy will transform in heat in the resistor.

If there is a circuit with a source of driving tension, resistor, capacitor and inductor connected in series, U_{dr} and $U_{ind} = - LdI/cdt$ must be equal to the sum of the tensions on the resistor, RI, and on the condenser, q/C, and rearranging we have

$$U_{dr} = RI + q/C + LdI/cdt \qquad \text{with} \qquad q = \int_0^t Idt. \tag{19.21}$$

The solution of this differential equation for a harmonic driving tension is given in Sect. 54.2 and I show then that it obviously violates the energy conservation.

At the end of this section let me give the formulas for the resistance, capacitance and inductance of two resistors, capacitors and inductors connected:

In series: $R = R_1 + R_2, \qquad 1/C = 1/C_1 + 1/C_2, \qquad L = L_1 + L_2,$ (19.22)

In parallel: $1/R = 1/R_1 + 1/R_2, \qquad C = C_1 + C_2, \qquad 1/L = 1/L_1 + 1/L_2.$ (19.23)

Indeed:

1) For two resistances in series we have $U = U_1 + U_2$, i.e., $RI = R_1 I + R_2 I$, and

for two resistance in parallel we have $I = I_1 + I_2$, i.e., $U/R = U/R_1 + U/R_2$.

2) For two condensers in series we have $U = U_1 + U_2$, i.e., $U/C = U/C_1 + U/C_2$, as the charges on condensers in series are equal, and for two condensers in parallel we have $q = q_1 + q_2$, i.e., $CU = C_1U + C_2U$, as the tensions on two condensers in parallel are equal.

3) For two inductors in series we have $U = U_1 + U_2$, i.e., $-LdI/dt = - L_1dI/dt - L_2dI/dt$, and for two inductors in parallel we have $I = I_1 + I_2$, i.e., $U/\omega L = U/\omega L_1 + U/\omega L_2$, where ω is the frequency of the alternating current (see Sect. 54.2).

20. DIELECTRICS AND MAGNETICS

20.1. DIELECTRICS.

Any medium is current conducting but the differences in the conductivities of the different media may be very large. The media with high conductivity are called CONDUCTORS, with low conductivity INSULATORS (or DIELECTRICS) and with medium conductivity SEMI-CONDUCTORS.

If a conductor is placed in an electric field, its side which points along the field will become charged positively and the opposite side, pointing against the field, negatively. This effect is called ELECTRIC POLARIZATION BY INDUCTION (shortly INDUCTION POLARIZATION) or ELECTROSTATIC INDUCTION.

If a dielectric is placed in an electric field, it becomes also polarized. We call this kind of electrostatic induction DIELECTRIC (or MOLECULAR) POLARIZATION. The difference between these two kinds of polarization is that the positive (resp., negative) charges provoking the induction polarization can be taken away and the conductor will then remain charged as a whole negatively (resp., positively), while the "polarization charges" of a dielectric cannot be taken away, and we call them BOUND CHARGES. The induction polarization appears because the FREE CHARGES (electrons) of the conductor increase their concentration at one side of the body and decrease it at the opposite side in an external electric field, while the dielectric polarization appears because the molecules of the dielectric become polarized, i.e., the one end of the molecule becomes positive and the other end negative (the molecules of certain media can always be polarized but they arrange themselves along a definite direction only in an external electric field.

The physical essence of the molecular polarization as well as the physical essence of the conduction of current are not clear enough.

Further only the dielectric polarization will be considered.

Let us have a parallel plate condenser between whose plates a dielectric is placed. When applying to the condenser a certain external tension U, on the left of its plates N positive charges will appear and on the right N negative charges. After the polarization of the dielectric (which appears with a certain very short retardation), on the left side of the dielectric $N - \Delta N$ negative charges will appear and on its right side N - ΔN positive charges. The negative bound charges on the left dielectric's surface will attract by induction other positive charges from the positive electrode of the source of driving tension and the charge on the left condenser's plate will increase, causing further increase of the bound charges on the left dielectric's surface. This process will go on until an equilibrium will be installed (the same appears on the right plate of the condenser). At the equilibrium state there will be $4\pi\chi N$ negative charges on the left dielectric's surface and $N + 4\pi\chi N = N(1 + 4\pi\chi)$ positive charges on the left condenser's plate, where χ is called ELECTRIC SUSCEPTIBILITY of the dielectric and

$$\varepsilon = 1 + 4\pi\chi \tag{20.1}$$

is called PERMITTIVITY of the dielectric (in the system SI one writes $\varepsilon = 1 + \chi$).

Now the electric intensity generated by the charges on the condenser's plates, called ELECTRIC DISPLACEMENT, will be

$$D = \varepsilon E = (1 + 4\pi\chi)E = E + 4\pi P, \tag{20.2}$$

where

$$P = \chi E \tag{20.3}$$

is called ELECTRIC POLARIZATION of the dielectric and it is $1/4\pi$ part of the electric intensity generated by the bound electric charges on the right and left surfaces of the dielectric.

The tension acting on the condenser $U = E.d$ (d is the distance between the condenser's plates) before putting the dielectric and after putting it is the <u>same</u>, thus the electric intensity between the plates also remains the <u>same</u>, E, and it is the <u>sum</u> of the electric intensity D produced by the charges on the condenser's plates and the electric intensity $-4\pi\chi E = -4\pi P$ produced by the bound charges on the left and right surfaces of the dielectric. Thus the <u>physically right equation</u> is not equation (20.2) but the following one

$$E = D - 4\pi\chi E = D - 4\pi P. \tag{20.4}$$

The electric displacement D <u>cannot</u> be measured. One can measure only the electric intensity E by making, for example, a narrow cut in the dielectric of the condenser and by putting there the measuring instrument.

20.2 MAGNETICS.

An inductor along which current flows is called ELECTROMAGNET (or shortly MAGNET). A solenoid is the most simple magnet. The centers of the solenoid's end windings are called POLES. NORTH POLE is the one from which one sees the current in the windings flowing counter-clockwise, and SOUTH POLE is the one from which one sees the current flowing clockwise. A small magnet is called also MAGNETIC DIPOLE.

According to the older concepts, the molecules of the media are magnetic dipoles. Usually these dipoles are pointing chaotically in all space directions. When put in an external magnetic field B, the magnetic dipoles arrange themselves along the field and the medium becomes magnet as a whole. The molecules may be not magnetic dipoles but they can become such only when the medium is put in an external magnetic field. This effect is called MAGNETIZATION and magnetizable medium is called MAGNETIC.

According to the now-a-day concepts not the whole molecule is a magnetic dipole but only the electrons are such magnetic dipoles with a strictly determined dipole moment and a strictly defined angular momentum, called SPIN, which is parallel to the magnetic dipole moment. When a magnetic is put in an external magnetic field those are the magnetic dipole moments of the electrons which arrange themselves along the field and so the magnetic becomes a magnet.

Let us put a magnetic in a long solenoid whose magnetic intensity is $\mathbf{B} = (4\pi nI/c)\hat{z}$ (see formula (18.28)). The magnetic field produced by the magnetic after its magnetization in the solenoid (which appears with a certain time retardation, especially when the magnetic goes out of the solenoid - see the Ewing effect in Sect. 54.5) is

$$4\pi\mathbf{M} = 4\pi\chi_m\mathbf{B}, \qquad (20.5)$$

where $\mathbf{M}$ is called MAGNETIZATION of the magnetic (it is equal to $1/4\pi$ part of the magnetic intensity produced by the magnetic) and χ_m is called MAGNETIC SUSCEPTIBILITY.

The resultant magnetic intensity in the solenoid will be

$$\mathbf{B}_\mu = \mathbf{B} + 4\pi\mathbf{M} = (1 + 4\pi\chi_m)\mathbf{B} = \mu\mathbf{B} \qquad (20.6)$$

and

$$\mu = 1 + 4\pi\chi_m \qquad (20.7)$$

is called PERMEABILITY of the magnetic (in the system SI one writes $\mu = 1 + \chi_m$).

Thus the resultant magnetic intensity is the sum of the initial magnetic intensity $\mathbf{B}$ and the magnetic intensity (20.5) produced by the magnetized magnetic, so that (20.6) is the physically right equation.

Usually one denotes the initial magnetic intensity by $\mathbf{H}$ and the symbol $\mathbf{B}$ is preserved for the final magnetic intensity when the magnetic is put in the electromagnet, calling it in this case MAGNETIC INDUCTION (or MAGNETIC FLUX DENSITY). With these notations equation (20.6) is to be written as follows

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M} = \mu\mathbf{H}. \qquad (20.8)$$

I am definitely against this separation. The magnetic intensity $\mathbf{H}$ and the "magnetic induction" $\mathbf{B}$ are not two different physical quantities. Whether in the solenoid there is a magnetic or another solenoid generating the same additional intensity $4\pi\mathbf{M} = 4\pi\chi_m\mathbf{B}$, there are absolutely no differences in the physical effects produced by these two systems. For this reason I shall very often use the word "magnetic intensity" both for $\mathbf{H}$ and $\mathbf{B}$, and often I shall use the symbol $\mathbf{B}$ for $\mathbf{H}$ and the symbol $\mathbf{B}_\mu$ for the "magnetic induction" $\mathbf{B}$, trying to emphasize in this way that between $\mathbf{B}$ and

H there is no principal physical difference.

The most tragic thing is that in the measuring system SI H and B are measured in different measuring units. For this reason this system must never be used in theoretical considerations when one wishes to understand the physical essence of the effects in electromagnetism.

And I should like to note that there is a substantial difference between dielectrics and magnetics. The dielectrics make only a new distribution of the available electric intensity, while the magnetics generate new magnetic intensity. As I already said, if one will cut a narrow slot in the dielectric of a parallel plate condenser, one will measure exactly the same electric intensity E which one will measure at the same point if there is no dielectric. However if one will cut a narrow slot in the magnetic of a solenoid, one will measure a μ times higher magnetic intensity than in the case where there is no magnetic. Thus the characters of dielectrics and magnetics are totally different and those who try to present electric polarization and magnetization as two similar phenomena do a big harm.

If $\chi_m < 0$, the MEDIUM is called DIAMAGNETIC, if $\chi_m = 0$, the medium is called NON-MAGNETIC, if $\chi_m > 0$, the medium is called PARAMAGNETIC and if $\chi_m \gg 0$, the medium is called FERROMAGNETIC.

The magnetic induction B in ferromagnetic materials depends not only on H but also on the "hystory", i.e., on the magnetic intensities which have acted on the material before putting it in the field of the magnetic intensity H. The dependence of B on the "hystorical" H (fig. 3) is called HYSTERESIS.

Let at the intial moment the ferromagnetic material be not magnetized. Thus if

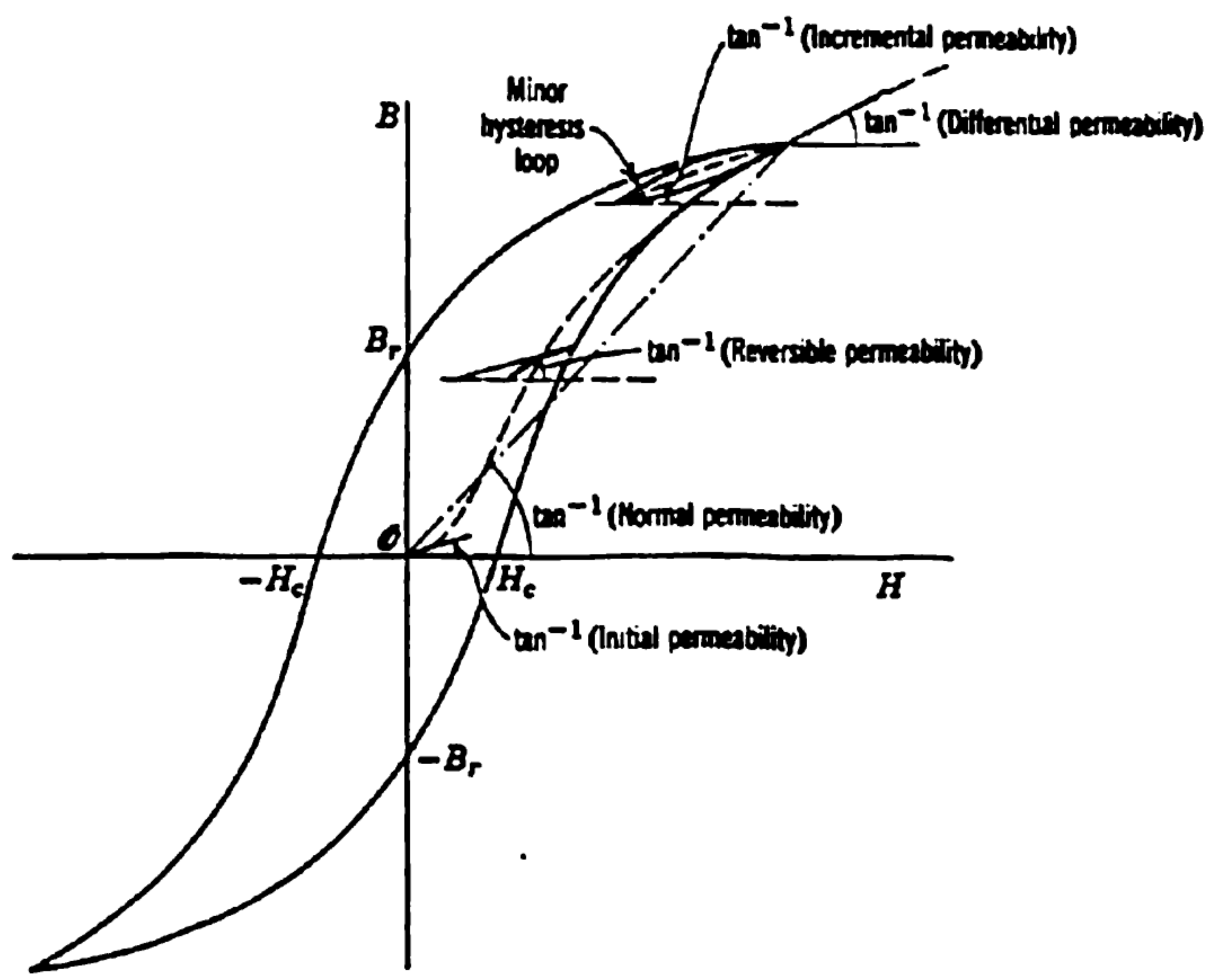

Fig. 3. The hysteresis loop.

the external magnetic intensity H is zero, the magnetic induction B produced by the magnetic will be zero. If H will begin to increase positively, B will also begin to increase positively and the dependence B = f(H) will be presented by the dashed line which begins from point 0. After coming to some maximum magnetic intensity H_{max}, let begin to diminish H. When coming at H = 0, the magnetic induction produced by the magnetic will be B_r and is called RESIDUAL (or REMANENT) MAGNETIC INDUCTION. After changing the direction of the magnetic intensity and letting it increase negatively, we shall arrive at the intensity $-H_c$ when the magnetic induction produced by the magnetic will be zero. $|-H_c|$ is called COERCIVE MAGNETIC INTENSITY (one says wrongly "COERCIVE FORCE"). After coming to $-H_{max}$ and returning again to H_{max}, we shall describe the closed loop in fig. 3 which is called the HYSTERESIS LOOP.

Let me note that there is "hysteresis" also at the polarization of dielectrics. Magnetics with large residual magnetic induction are called PERMANENT MAGNETS (shortly MAGNETS) and dielectrics with large residual electric displacement are called ELECTRETS.

In fig. 3 there are shown different kinds of permeabilities defined by the relation

$$\mu = \arctan(B/H), \qquad (20.9)$$

noting that in the figure the "arctan" is designated by "$\tan^{-1}$".

It can be shown that the area of the hysteresis loop in fig. 3 is equal to the energy which is lost in the form of heat for magnetizing, demagnatizing, anti-magnetizing, demagnetizing and again magnetizing of unity volume of the magnetic. This energy is called HYSTERESIS LOSSES. The effect is no more a pure electromagnetic effect as heat becomes involved.

Let consider now a <u>closed</u> magnetic with length L and cross-section S, whose permeability is μ. If a coil with N turns is wound on it in which current I flows, this is called a TORUS. The most simple torus is the circular one, with radius R and radius of the turns $r = \sqrt{S/\pi}$. For $R \gg r$ the magnetic intensity in the torus is as in a very long solenoid (see (18.28)) $H = 4\pi NI/cL$ and the magnetic induction is $B = 4\pi\mu nI/c$, where $n = N/L$ is the number of the windings on a unit of length. If not the whole length of the torus is covered by the N turns but only a certain part ΔL of it and μ is high enough, the magnetic induction in the iron will be $B = 4\pi\mu nI/c$, where now $n = N/\Delta L$. The iron on which the coil is wound is called CORE, and the iron which "conducts" the magnetic flux and closes it is called YOKE.

I introduce the notion MAGNETIC TENSION (official physics calls it "MAGNETOMOVING FORCE"), U_m, as folows

$$U_m = (4\pi/c)NI = (4\pi/c)nIL = HL = (B/\mu)L. \qquad (20.10)$$

If μ does not remain constant in the whole torus, we shall have

$$U_m = \oint_L (B/\mu)dL = \oint_L (\Phi/\mu S)dL = \Phi\oint_L dL/\mu S = \Phi R_m \qquad (20.11)$$

This equation has a form similar to that of Ohm's law (16.5). Here the magnetic tension U_m stays for the electric tension U, the magnetic flux Φ stays for the electric current I and the "magnetic resistance" R_m, called RELUCTANCE, stays for the electric resistance R. The analogy between Ohm's law in electricity (16.5) and "Ohm's law in magnetism" (20.11) is purely formal and has no certain physical background.

The quantity reciprocal to R_m

$$G_m = 1/R_m = \mu S/L \qquad (20.12)$$

is called PERMEANCE. Thus permeability μ corresponds to the conductivity γ (see (16.3)).

Let have a slot of small length l in the iron ring, and let us assume that the magnetic flux remains constant along the whole length of the torus, i.e., let us assume that there is no dispersion of magnetic flux in the slot.

Now we shall have for the reluctance, according to the last part of equation (20.11),

$$R_m = (L - l)/\mu S + l/S = \{L + l(\mu-1)\}/\mu S \cong (L + \mu l)/\mu S. \qquad (20.13).$$

Thus an air slot of length l increases the reluctance as an additional iron part of length $L' = \mu l$.

21. THE DIFFERENT KINDS OF ELECTRIC INTENSITY

According to the concepts of official physics, which I shall call the first concepts, the EFFECTS on charges at rest are called ELECTRIC and the effects on charges in motion are called MAGNETIC. I also followed these concepts when separating the terms in the Newton-Lorentz equation (8.5) into two electric terms, presented under the common name "restricted electric intensity", and into two magnetic terms, the vector magnetic intensity and the scalar magnetic intensity (official physics, of course, ignores the latter).

However the separation of the effects into electric and magnetic can be done following other second concepts, namely, considering as electric the effects due to the action of charges at rest and as magnetic those due to the action of charges in motion. Now only the first term in the Newton-Lorentz equation (8.5) will be called electric and the other three terms magnetic, although the fourth term, in view of equation (8.8) can be considered as electric and as magnetic, noting, however, that to have $\partial\Phi/\partial t \neq 0$, the charges must move.

Both these separations of the effects in electromagnetism into electric and magnetic have their positive and negative aspects and the best way is to consider all effects as common ELECTROMAGNETIC EFFECTS. In these third concepts, however, it is convenient to give to the notion "electric" the pedominance and to try to evade as much as possible the notion "magnetic".

Following these third concepts, I called the net force acting on a test charge "global electric intensity". I give to the different parts of this force

$$E_{coul} = - \text{grad}\Phi, \quad E_{tr} = - \partial \mathbf{A}/c\partial t, \quad E_{mot} = (\mathbf{v}/c)\times \text{rot}\mathbf{A}, \quad E_{whit} = - (\mathbf{v}/c)\text{div}\mathbf{A} \quad (21.1)$$

the names: COULOMB ELECTRIC INTENSITY, TRANSFORMER ELECTRIC INTENSITY, MOTIONAL ELECTRIC INTENSITY and WHITTAKER ELECTRIC INTENSITY.

The transformer electric intensity can have two substantially different aspects:

a) REST-TRANSFORMER ELECTRIC INTENSITY (in case where the wires of the surrounding system are at rest and only the flowing currents change)

$$E_{rest-tr} = - (1/c)\partial \mathbf{A}/\partial t. \quad (21.2)$$

b) MOTIONAL-TRANSFORMER ELECTRIC INTENSITY (in case where the currents flowing in the wires of the surrounding system are constant but the wires move, and the magnetic potential becomes a composite function of time through the radius vectors $\mathbf{r}_i$ connecting the different current elements with the reference point)

$$E_{mot-tr} = - \frac{1}{c}\sum_{i=1}^{n} \frac{\partial \mathbf{A}_i\{\mathbf{r}_i(t)\}}{\partial t} = - \frac{1}{c}\sum_{i=1}^{n} \left(\frac{\partial \mathbf{A}_i}{\partial x}\frac{\partial x_i}{\partial t} + \frac{\partial \mathbf{A}_i}{\partial y}\frac{\partial y_i}{\partial t} + \frac{\partial \mathbf{A}_i}{\partial z}\frac{\partial z_i}{\partial t}\right) = \frac{1}{c}\sum_{i=1}^{n}(\mathbf{v}_i.\text{grad})\mathbf{A}_i \quad (21.3)$$

where $\mathbf{v}_i = - \partial \mathbf{r}_i/\partial t$ is the velocity of the i-th current element of the surrounding system which generates the magnetic potential $\mathbf{A}_i$ at the reference point. The time derivative of the radius vector $\mathbf{r}_i$ is taken with a negative sign, as $\mathbf{r}_i$ points from the i-th current element to the reference point. If the surrounding system, i.e., the magnet, moves translatory, we shall have $\mathbf{v}_i = \mathbf{v}$ and thus

$$E_{mot-tr} = (1/c)(\mathbf{v}.\text{grad})\mathbf{A}. \quad (21.4)$$

The motional-transformer electric intensity and the formula describing it were discovered by me(6), although every child must come to this "discovery" following the elementary mathematical logic. I repeat once more (see Sect. 14) that in electromagnetism there are only three discoveries: the law of Coulomb, Neumann and Newton (i.e., Newton's law for gravitational energy of two masses leading to the world gravitational energy of mass m, U_w, which when taken with negative sing gives the time energy of m, e_0). All other electromagnetic "effects" are simple logical conclusions to which these three laws lead, after introducing the most simple models for conductors, dielectrics and magnetics.

Why then official physics does not know the motional-transformer electric intensity? The answer is: Because of the introduction in physics of the wrong PRINCIPLE OF RELATIVITY. Indeed, according to this principle, all physical effects must depend only on the relative velocities of the bodies. Thus, this principle asserts that if at a motion of a wire with velocity $\mathbf{v}$ respectively to a magnet at rest the induced in the wire electric intensity is given by the third formula (21.1), then the electric intensity induced in the wire at rest when the magnet moves with a velocity $\mathbf{v}$ will be

$$E_{relativistic} = - E_{mot} = - (1/c)\mathbf{v}\times \text{rot}\mathbf{A}. \quad (21.5)$$

How many papers and books have been written to show that the stupidity (21.5) must be true!

Let me present here the experiemnt of Kennard[14] which in my simplified variation (fig. 4) was labeled by J. Maddox[15] as "Stefan Marinov's puzzle". As a matter of fact, there is no puzzle at all, as Kennard's experiment is a trivial illustration of the difference between the motional and motional-transformer electric intensities and the "puzzle" is only in the heads of the poor relativists.

I shall present first the description of the puzzle by John Maddox' own words:[15]

> ... from time to time, in Marinov's copious writings, there are relatively simple arguments that appear accessible even to those still at high school. Here is one series of *gedanken* experiments presented as if it were a Christmas puzzle (the original intension), with some helpful (or misleading) hints for its solution.
>
> The figure (fig. 4) shows a pair of circular conductors arranged as two concentric circles. Equal electric currents are circulated in each, but in opposite directions. The simplest way of creating this arrangement is to cut through the concentric pair at some point and to join the loose ends in pairs by short lengths of straight conductor. An electromotive force applied anywhere along the conductor will engender a current which must be everywhere uniform. At the bridged gap, there will be equal currents flowing in opposite directions, so their influence on the magnetic fields in the concentric gap will be zero.
>
> The device is thus a means of arranging that there is a uniform magnetic field in the space between the concentric circles in a direction perpendicular to their plane (downwards into the plane of the paper when the current in the circuit flows in the direction indicated). The sensor in the experiment is a conductor long enough just to bridge the gap between the concentric circles and mounted on thin insulating support in such a way that it can be made to slide around the circle. The objective is to measure the voltage across the sliding conductor, either by a standard voltmeter or by a condenser whose accumulated charge will be a measure of the voltage in a steady state.

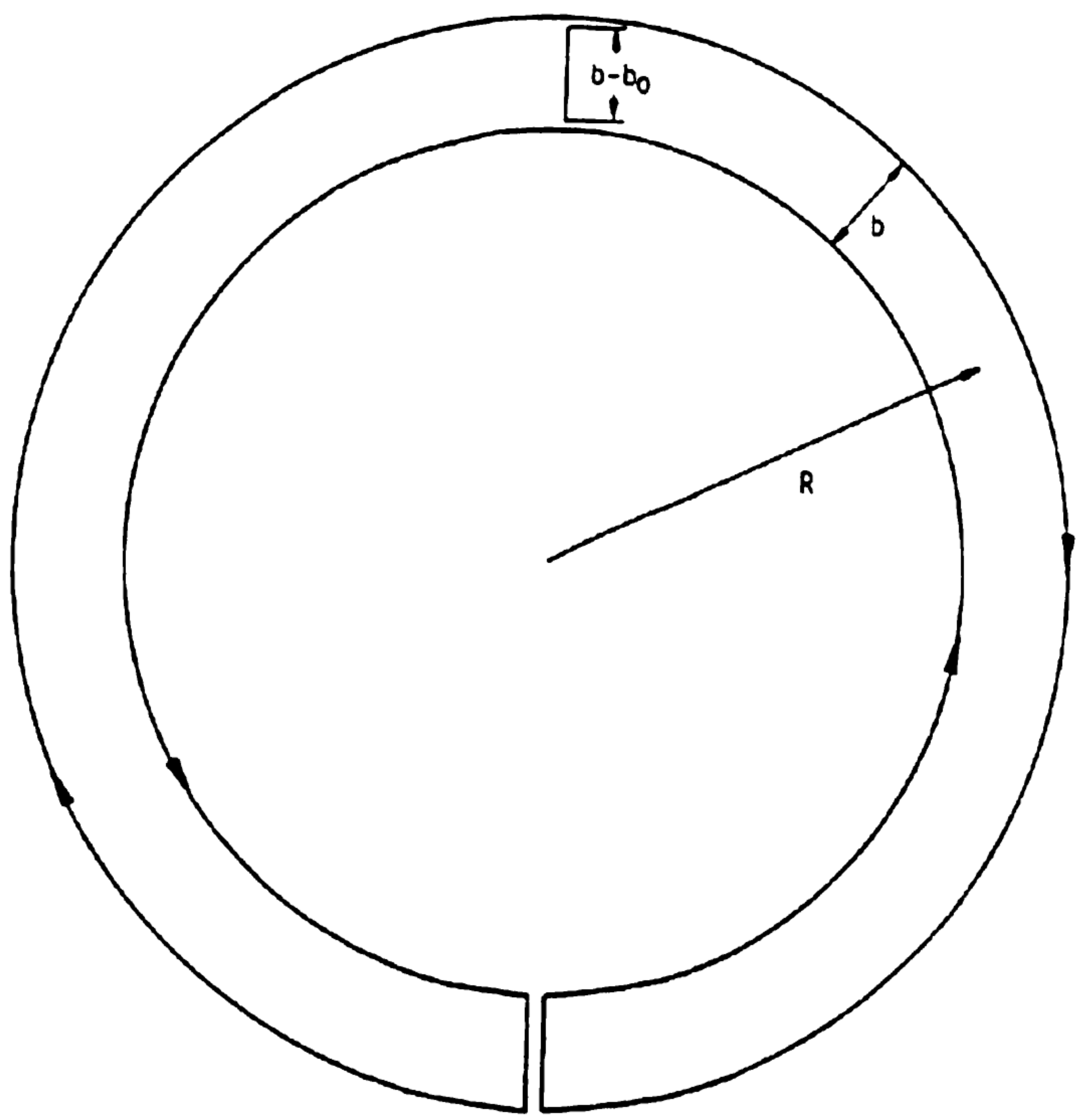

Fig. 4. Kennard's experiment.

The simplest case is when the sliding conductor is at rest. Then there is no voltage. Right? Next comes the case in which the sliding conductor moves at uniform speed around the concentric gap, always pointing along a radius of the concentric circles. As the slider moves, it will cut through magnetic lines of force at constant rate, so that there will be a constant voltage across the ends. The polarity of the slider will depend only on the direction of the current in the concentric circuit, and not on whether the slider moves clockwise or anticlockwise. Right again?

Now come the tricky part, at least so far as Marinov is concerned. What happens if the sliding conductor is fixed in space, but the undrlying concentric circuit is rotated about its center? Relativity theory naturally predicts that the voltage across the sliding conductor would be the same as in the first experiment, and with the same polarity. On the other hand, questions may be raised about the degree to which the pattern of magnetic forces generated by the current is dragged around the ring by its rotation. Maybe there is a smaller voltage, but with the same polarity. What, asks Marinov, is the answer?

The second conundrum is superficially simpler: simply rotate the apparatus in its own plane, about the center of the concentric circles. (There will be a small voltage due to Earth's magnetic field, but this may safely be neglected.) Is there now a voltage, and with what polarity? If the answer to the first question is "Yes" the answer to the second must be "No", and vice versa. Readers are invited to make up their minds before reading on.

Marinov's own answers are unambiguous. Vice versa wins the day. When the underlying concentrc circle is rotated and the slider is kept fixed, there is no voltage across the movable conductor. But when the whole apparatus is rotated about its centre, the voltage across the now-moving sliding conductor is identical with that obtained when the slider is moving relative to the concentric circuit.

The implications are evidently important. The null answer to Marinov's first question implies that relativity has vanished through the window, the affirmative answer to the second implies that an isolated apparatus carrying a circulating current will generate a voltage when rotated, which raises forbidden questions about absolute space.

Here are my comments:

First about Maddox' language:

1) For "electric tension" Maddox (and whole official physics) uses the word "voltage". But if following such a trend, we have to call the current "amperage", the magnetic intensity "teslage", etc.

2) For "driving tension" Maddox (and whole official physics) uses the very bad word "electromotive force".

Then about Maddox' concepts: To speak at the end of the XXth century about "MAGNETIC FORCE LINES" and to ruminate (as Faraday did) whether these lines will move when a current wire producing them moves is the same thing as at the end of XXth century to ride a horse on London's Strand. In electromagnetism there are only charges, moving charges (i.e., current elements), distances and a watch on the physicist's left hand. And nothing else!

Finally about three Maddox' obvious errors, the first one being an essential error and the two other *lapsus calamiti*:

1) The tension along the slider can be measured only by the help of a condenser which accumulates the charges generated at its ends and by leading them to an electrometer, as KENNARD did in his EXPERIMENT.[14]. In my quasi-Kennard experiment (see fig. 5 and Sect. 45) the availability of charges at the ends of the slider was

indicated electrometrically directly by "golden leaves". By the help of a "standard voltmeter" the difference between the motional and motional-transformer induced electric tensions cannot be demonstrated, as at the ends of the slider one must put sliding contacts and at motion of the voltmeter with its wires leading to the sliding contacts a tension will be induced in these wires exactly equal and opposite to the tension induced in the slider when it moves with the same velocity.

2) Maddox writes that the polarity on the slider will not depend on whether the slider moves clockwise or anti-clockwise. This is wrong. By changing the sense of the slider's rotation the polarity of the tension induced in the slider will also change.

3) Maddox writes that the two concentric current wires generate a "uniform" magnetic field. This is not true. The magnetic field is <u>not</u> uniform. It is the strongest near the concentric wires and the weakest along the middle circle between them.

Now I shall calculate the effects in Maddox' "puzzle" which is not at all a puzzle but, as already said, a trivial illustration of the third formula (21.1) and of formula (21.4).

To be able to make these calculations, let us find first the magnetic potential generated by two currents I flowing in two infinitely long parallel wires separated by a distance b. In fig. 5 two such wires are presented assuming that their lengths, d, tend to infinity. If the frame's origin is taken at the center of the rectangle, the ordinate of the upper wire will be b/2 and of the lower - b/2. The current in the rectangular loop in fig. 5 is flowing in positive, i.e., anti-clockwise direction, thus in a direction opposite to the current's direction in fig. 4.

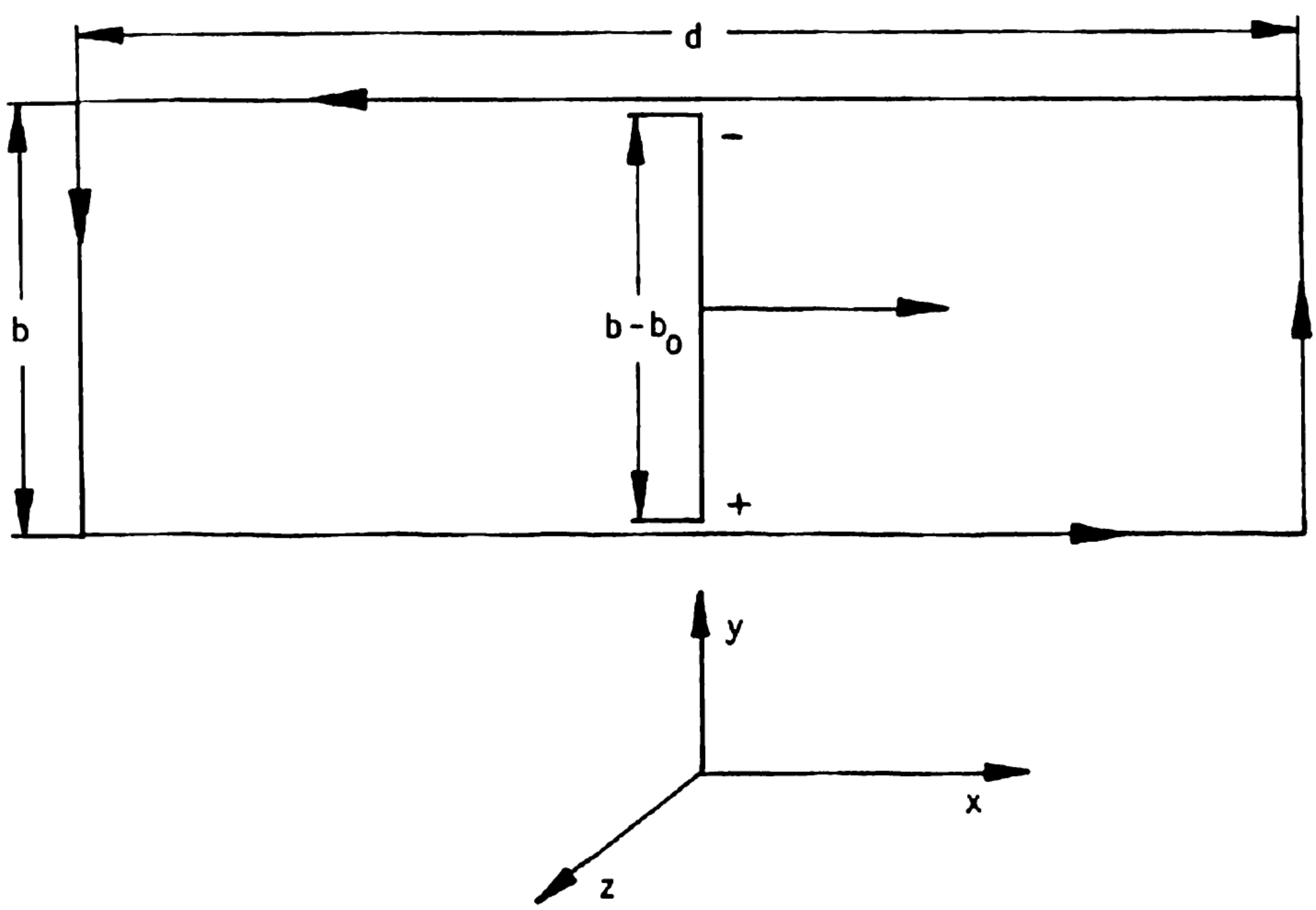

Fig. 5. The quasi-Kennard experiment.

According to formula (18.15) we shall have for the x-component of **A** generated by the upper wire at a reference point taken on the y-axis

$$A_x = -(I/c)\int_{-d/2}^{d/2}\{(b/2-y)^2+x^2\}^{-1/2}dx = -(2I/c)\ln\frac{d/2+\{(b/2-y)^2+d^2/4\}^{1/2}}{b/2-y}, \quad (21.6)$$

the components A_y and A_z being equal to zero. We see that for $d\to\infty$ the component A_x tends to infinity. However the magnetic potential generated by the upper and lower currents in fig. 5 is final also for infinitely long wires, namely

$$A_x = -\frac{2I}{c}\ln\frac{d/2+\{(b/2-y)^2+d^2/4\}^{1/2}}{b/2-y} + \frac{2I}{c}\ln\frac{d/2+\{(b/2+y)^2+d^2/4\}^{1/2}}{b/2+y} \cong$$

$$\frac{2I}{c}\ln\frac{b/2-y}{b/2+y}, \quad (21.7)$$

where the result on the right side is written for d long enough and y can take any value except b/2 and - b/2.

These two long d-wires can be connected with the short b-wires and so we shall obtain a rectangular loop with $d \gg b$. As the two b-wires are far enough from the reference point, their contribution to the magnetic potential can be neglected.

I shall calculate the effects for the rectangular long loop in fig. 5. If the radius R in fig. 4 is large enough, i.e., if $R \gg b$, the same effects will be valid also for the concentric loops in fig. 4.

The magnetic intensity for reference points along the y-axis will be if using formula (21.7)

$$\mathbf{B} = \text{rot}\mathbf{A} = -(\partial A_x/\partial y)\hat{z} = \frac{8Ib\hat{z}}{c(b^2-4y^2)} \quad (21.8)$$

and the electric intensity induced along the moving slider will be

$$\mathbf{E}_{mot} = \mathbf{v}\times\mathbf{B}/c = (vB_z/c)\hat{y} = \frac{8vIb\,\hat{y}}{c^2(b^2-4y^2)} \quad (21.9)$$

For the electric tension induced along the slider with length $b-b_0$ we shall have

$$U_{mot} = \int_{-b/2+b_0/2}^{b/2-b_0/2}(E_{mot})_y dy = (4vI/c^2)\text{Artanh}(2y/b)\Big|_{-(b-b_0)/2}^{(b-b_0)/2} =$$

$$(4vI/c^2)(1/2)\ln\frac{1+2y/b}{1-2y/b}\Big|_{-(b-b_0)/2}^{(b-b_0)/2} = (4vI/c^2)\ln\frac{2b-b_0}{b_0} \cong (4vI/c^2)\ln(2b/b_0), \quad (21.10)$$

where the result on the left is for $b \gg b_0$.

Meanwhile we shall have for the electric intensity induced in the slider at rest when the long rectangular loop in fig. 5 moves with velocity **v**

$$\mathbf{E}_{mot-tr} = (\mathbf{v}.\text{grad})\mathbf{A}/c = (v/c)\partial\mathbf{A}/\partial x = 0. \quad (21.11)$$

When moving both the slider and the rectangular loop in fig. 5 with a velocity **v** the electric intensity induced in the slider will be the sum of the motional (21.9) and motional-transformer (21.11) intensities, thus the tension induced will be given by formula (21.10). That's the whole "puzzle" of Dr. Maddox and the relativity blind.

Let me note that the magnetic intensity produced by a very long wire at a distance r, according to formula (21.8), in which we put b/2 = r, y = 0, will be

$$B_{single} = (1/2)B_{double} = 2I/cr. \qquad (21.12)$$

The electric intensities (21.1) are the kinetic forces of the unit test charge. They can lead to the motion of the test charge in the conductor, and in such a case we call them ELECTROMOTIVE FORCES or they can be transferred from the charge on the metal lattice (ions' lattice) setting the whole conductor in motion, and in such a case we call them PONDEROMOTIVE FORCES. All four electric intensities (21.1) can lead to electromotive forces but only E_{mot} and E_{whit} can lead to ponderomotive forces. When v is the velocity of the test charge in the conductor, E_{mot} and E_{whit} generate ponderomotive forces, and when v is the velocity of the conductor, E_{mot} and E_{whit} generate electromotive forces. If E_{coul} and E_{tr} have pushed the charges to the extremities of the conductor and for them there is no more motional freedom, E_{coul} and E_{tr} can also generate ponderomotive forces.

The phenomenon of induction of electric intensity in conductors (and dielectrics) is called ELECTROMAGNETIC INDUCTION. The electromagnetic induction described by the third formula (21.1) is called MOTIONAL INDUCTION, by the fourth formula (21.1) WHITTAKER INDUCTION, by formula (21.7) REST-TRANSFORMER INDUCTION and by formula (21.3) MOTIONAL-TRANSFORMER INDUCTION. The induction of electric intensity in conductors (and dielectrics) according to the first formula (21.1) was called (see Sect. 20.1) ELECTROSTATIC INDUCTION.

Now I shall point out at the reason which has not allowed to humanity, during two centuries of experimental work, to reveal the difference between the motional and motional-transformer inductions.

The reason is that for closed loops the induced motional and motional-transformer electric tensions are equal with opposite signs. Indeed, we have for the tensions induced in a closed loop for the case where loop and magnet will be moved with a velocity **v** together in the laboratory

$$c(U_{mot} + U_{mot-tr}) = \oint_L (\mathbf{v}\times rot\mathbf{A}).d\mathbf{r} + \oint_L \{(\mathbf{v}.grad)\mathbf{A}\}.d\mathbf{r} = \int_S rot\{\mathbf{v}\times rot\mathbf{A} + (\mathbf{v}.grad)\mathbf{A}\}.d\mathbf{S} = 0, \qquad (21.13)$$

where S is an arbitrary surface spanned over the loop L, and taking into account formula (7.10) and the mathematical rule that rot(grad) of any scalar function is equal to zero, we conclude that the surface integral is identically equal to zero. Thus we obtain

$$U_{mot} = - U_{mot-tr}. \qquad (21.14)$$

Proceeding from this equation which is not generally valid but only for closed loops Einstein created the monster called "theory of relativity" (see his 1905-paper).

22. THE POTENTIALS, NOT THE INTENSITIES, DETERMINE THE ELECTROMAGNETIC EFFECTS

The childishly simple theory obtained when proceeding from the axiomatic Coulomb, Neumann and Newton laws asserts that the electromagnetic effects are determined by the electric and magnetic potentials. Official physics asserts that the electromagnetic effects are determined by the electric and magnetic intensities (of course ignoring the scalar magnetic intensity).

The intensities are space and time derivatives of the potentials and, of course, they will also determine the electromagnetic effects. But as any derivative carries less information than the function itself, so the intensities may not be able to explain all effects which are described in all details by the potentials.

In my theory, if a material system is given, then the electric and magnetic potentials are uniquely defined by the help of the definition equalities (8.1). Thus the potentials Φ and $\mathbf{A}$ are the primordial quantities which determine the motion of the test charge. According to official physics, the primordial quantities which determine the motion of the test charge are the restricted electric intensity $\mathbf{E}$ and the vector magnetic intensity $\mathbf{B}$. Thus for official physics any two potentials Φ, $\mathbf{A}$ which, when put in the first two equations (8.6) give the right intensities $\mathbf{E}$, $\mathbf{B}$, have the whole right to be treated as potentials of the system in consideration.

Let us have two potentials Φ, $\mathbf{A}$ which give the right intensities $\mathbf{E}$, $\mathbf{B}$. Let us take an arbitrary function $f(\mathbf{r},t) = f(x,y,z,t)$ of the radius vector of the reference point and of time and write two "new" potentials

$$\Phi' = \Phi - \partial f/\partial t, \qquad \mathbf{A}' = \mathbf{A} + \mathrm{grad} f. \tag{22.1}$$

If putting Φ' and $\mathbf{A}'$ in the first two equations (8.6), we shall obtain two new intensities

$$\mathbf{E}' = -\,\mathrm{grad}(\Phi - \partial f/\partial t) - (\partial/c\partial t)(\mathbf{A} + \mathrm{grad} f) = -\,\mathrm{grad}\Phi - \partial \mathbf{A}/c\partial t = \mathbf{E},$$

$$\mathbf{B}' = \mathrm{rot}(\mathbf{A} + \mathrm{grad} f) = \mathrm{rot}\mathbf{A} + \mathrm{rot}(\mathrm{grad} f) = \mathrm{rot}\mathbf{A} = \mathbf{B}. \tag{22.2}$$

It turns thus out that the new intensities are identical with the old ones. And according to official physics the new potentials have the same right to be considered as potentials of the system in consideration. Official physics calls the transformation (22.1) GAUGE TRANSFORMATION and the function $f(\mathbf{r},t)$ GAUGE TRANSFORMATION FUNCTION.

So, according to official physics, one can take as a gauge transformation function the following one

$$\partial f/c\partial t = \Phi, \tag{22.3}$$

obtaining thus the new electric potential equal to zero in whole space. Taking into account also the equation of potential connection (8.8), we shall thus have

$$\Phi' = 0, \qquad \mathrm{div}\mathbf{A}' = 0. \tag{22.4}$$

Official physics considers thus as justified to erase the reality of the electric and scalar magnetic fields. Monstruous!

For my theory (and for the Divinity) the gauge transformation (22.1) is inadmissible and not the intensities but the potentials determine thoroughly the effects in electromagnetism.

Now I shall show with simple considerations how the gauge transformation (22.1) may lead to contradictions with the physical reality.

In Sect. 18 I have calculated **A** and B of a very long circular solenoid. Now I shall do this for a very long solenoid with rectangular cross-sectuon.

As the exact calculation is pretty complicated (I have not seen such a calculation in the literature!), I shall present here a very simple approximate calculation which also leads to the right result.

Formula (21.7) gives the magnetic potential generated by the rectangular loop shown in fig. 5 at the assumtion d >> b. Let us now suppose that there are n such loops on a unit of length along the z-axis going from $z = -\infty$ to $z = \infty$. As in such a case there will be ndz turns along the differential length dz, the resultant magnetic potential is to be calculated according to the following formula, if we shall suppose $b \gg |y|$, i.e., if we shall suppose that the reference point is near to the x-axis,

$$A_x = \frac{2I}{c}\int_{-\infty}^{\infty}\ln\left\{\frac{(b/2-y)^2+z^2}{(b/2+y)^2+z^2}\right\}^{1/2} n\,dz = \frac{I}{c}\int_{-\infty}^{\infty}\left\{\ln\left(1-\frac{by}{b^2/4+z^2}\right)-\ln\left(1+\frac{by}{b^2/4+z^2}\right)\right\}n\,dz =$$

$$-\frac{nI}{c}\int_{-\infty}^{\infty}\frac{2by}{b^2/4+z^2}\,dz = -\frac{4nIy}{c}\arctan(2z/b)\Big|_{-\infty}^{\infty} = -4\pi nIy/c, \qquad (22.5)$$

where I neglected y^2 with respect to $b^2/4$ and then I presented the logarithm as a power series neglecting the powers higher than the first.

For the magnetic intensity we obtain

$$\mathbf{B} = \mathrm{rot}\mathbf{A} = -(\partial A_x/\partial y)\hat{z} = (4\pi nI/c)\hat{z}, \text{ i.e., i.e., } \quad B_z = 4\pi nI/c. \qquad (22.6)$$

Thus the vector of the magnetic intensity in the rectangular very long solenoid will have the following Cartesian components

$$\mathbf{A}_{rect} = (-4\pi nIy/c,\ 0,\ 0) = (-yB_z,\ 0\ ,\ 0). \qquad (22.7)$$

According to formula (18.26), we shall have for **A** and B in a circular very long solenoid

$$A_\phi = 2\pi nI\rho/c, \qquad B_z = 4\pi nI/c. \qquad (22.8)$$

Thus the magnetic intensities in two very long solenoids with circular and rectangular cross-sections are equal. However the magnetic potentials are not. The magnetic potential in the long solenoid with prolongated rectangular cross-section is given by formula (22.7), while, taking into account that Cartesian components of the magnetic potential in the circular solenoid are $A_x = -A_\phi \sin\phi = -A_\phi y/\rho$, $A_y = A_\phi\cos\phi = A_\phi x/\rho$, we shall have

$$A_{circ} = (-2\pi nIy/c,\ 2\pi nIx/c,\ 0) = (-yB_z/2,\ xB_z/2,\ 0). \qquad (22.9)$$

The transformation from the potential (22.7) to the potential (22.9), or vice versa is, of course, a gauge transformation. Indeed, choosing the gauge transformation function in (22.1) in the form $f(x,y,z,t) = B_z xy/2$, we obtain the potential (22.9) if proceeding from the potential (22.7)

$$\mathbf{A}' = \mathbf{A} + \text{grad}f = -\ yB_z\hat{x} + (yB_z/2)\hat{x} + (xB_z/2)\hat{y} = -(yB_z/2)\hat{x} + (xB_z/2)\hat{y}. \qquad (22.10)$$

Thus, according to official physics, for magnetic potentials in two very long solenoids with circular and rectangular cross-sections (with $d \gg b$!) one can take both quantities (22.7) and (22.9) and all effects will be determined by the magnetic intensity B_z given in (22.6) and (22.8) which has the same value in both solenoids.

To show that this is not true, let us put an electric charge q at the centers of both solenoids. If moving this charge with a velocity v in both solenoids first along the x-axis and then along the y-axis, the acting force, of course, will be the same:

a) motion of the charge along the x-axis

$$f = qE_{mot} = (q/c)v\hat{x}\times B_z\hat{z} = -\ (qvB_z/c)\hat{y} = -\ (4\pi qvnI/c^2)\hat{y}, \qquad (22.11)$$

b) motion of the charge along the y-axis

$$f = qE_{mot} = (q/c)v\hat{y}\times B_z\hat{z} = (qvB_z/c)\hat{x} = (4\pi qvnI/c^2)\hat{x}. \qquad (22.12)$$

However if moving the solenoids with a velocity v, leaving the charge at rest, the acting force will be

a) motion of the solenoid with circular cross-section along the x-axis

$$f = (q/c)(v\hat{x}.\text{grad})(-yB_z\hat{x}/2 + xB_z\hat{y}/2) = (qvB_z/2c)\hat{y} = (2\pi qvnI/c^2)\hat{y}, \qquad (22.13)$$

a') motion of the solenoid with rectangular cross-section along the x-axis

$$f = (q/c)(v\hat{x}.\text{grad})(-yB_z\hat{x}) = 0, \qquad (22.14)$$

b) motion of the solenoid with circular cross-section along the y-axis

$$f = (q/c)(v\hat{y}.\text{grad})(-yB_z\hat{x}/2 + xB_z\hat{y}/2) = -\ (qvB_z/2c)\hat{x} = -\ (2\pi qvnI/c^2)\hat{x}, \qquad (22.15)$$

b') motion of the solenoid with the rectangular cross-section along the y-axis

$$f = (q/c)(v\hat{y}.\text{grad})(-yB_z\hat{x}) = -\ (qvB_z/c)\hat{x} = -\ (4\pi qvnI/c^2)\hat{x}. \qquad (22.16)$$

Thus the motion of the test charge in these two solenoid, at motion of the solenoids, will be completely different, although the magnetic intensities in the solenoids remain the same.

I should like to note that when calculating the integral (22.5) I integrated for z in the limits for $-\infty$ to ∞, while when calculating the integral (18.23) I integrated for z in the limits from 0 to ∞. Easily can be seen that if in (18.23) I had also calculated in the limits from $-\infty$ to ∞, a value for A two times than the right one should be obtained. I could not found an explanation for this discrepancy, noting that when B. B. Dasgupta (Am. J. Phys., 52, 258, (1984)) calculates directly the magnetic intensity in a long circular solenoid he integrates for z in the limits from $-\infty$ to ∞ and obtains the right result. Scott[12] (p.322) makes the calculation through the magnetic potential, exactly as I do; he takes z in the limits from $-\infty$ to ∞ but the result which he then writes is two times smaller than this one which is to be obtained at a right mathematical calculation. I turn the attention of the mathematicians to this strange discrepancy.

23. ABSOLUTE AND RELATIVE NEWTON-LORENTZ EQUATIONS

The Newton-Lorentz equation (8.4) is written in a frame attached to absolute space and I call it the ABSOLUTE NEWTON-LORENTZ EQUATION.

Let us now find the form of the Newton-Lorentz equation in a laboratory (frame) moving with a velocity $\mathbf{V}$ in absolute space, where it will be called the RELATIVE NEWTON-LORENTZ EQUATION, begging once more the reader to pay attention to the difference between the Lorentz and Marinov invariances considered in Sect. 1. Thus I shall look for the Newton-Lorentz equation not forthe system considered first with mass center at rest in absolute space and then with its mass center moving with velocity $\mathbf{V}$ in absolute space but if the observer would move with velocity $\mathbf{V}$ in absolute space and the system considered remains always with mass center at rest in absolute space.

Let the velocities of the test charge and of the charges of the system in consideration by $\mathbf{v}$ and $\mathbf{v}_i$ with respect to absolute space and $\mathbf{v}'$, $\mathbf{v}_i'$ with respect to the laboratory which moves with the velocity $\mathbf{V}$ in absolute space.

As the velocity of the moving laboratory can be not high (the velocity of a laboratory attached to the Earth is about 300 km/sec!), it is enough to use the Galilean formulas for the addition of velocites

$$\mathbf{v} = \mathbf{v}' + \mathbf{V}, \qquad \mathbf{v}_i = \mathbf{v}_i' + \mathbf{V}, \tag{23.1}$$

which can be obtained when differentiating formula (3.1) with respect to time (of course written in three dimensions), and not the Marinov formulas for addition of velocities which can be obtained [3,5] at the differentiation of formula (3.5).

Let me note that in Ref. 5 I consider the effects which can be observed if the mass center of the system in consideration (usually a single particle) is considered first at rest in absolute space and then moving with a velocity $\mathbf{v}$ in absolute space. In this case the velocity $\mathbf{v}$ can be high (even approaching c) and the Marinov or the Lorentz transformation formulas are to be used (I repeat - see Sect. 3 - when considered from an absolute point of view these two transformations lead to identical results).

Thus using (23.1), we shall have for the argument of the gradient in formula (8.3), having in mind the definition formulas for the potentials (8.1),

$$\Phi - \frac{\mathbf{v}.\mathbf{A}}{c} = \sum\frac{q_i}{r_i} - \frac{\mathbf{v}'+\mathbf{V}}{c}.\sum\frac{q_i(\mathbf{v}_i'+\mathbf{V})}{cr_i} = \Phi'(1 - \frac{\mathbf{v}'.\mathbf{V}}{c^2} - \frac{V^2}{c^2}) - \frac{\mathbf{v}'+\mathbf{V}}{c}.\mathbf{A}', \qquad (23.2)$$

where $\Phi' = \Phi$ is the relative electric potential which is equal to the absolute electric potential, as the electric potential is not velocity dependent, $\mathbf{A}' = \sum q_i \mathbf{v}_i'/cr_i$ is the relative magnetic potential, and the summations are taken over the n charges of the system in consideration.

The total time derivatives of the absolute and relative magnetic potentials must be equal

$$d\mathbf{A}/dt = d\mathbf{A}'/dt, \qquad (23.3)$$

because $d\mathbf{A}/dt$ depends only on the changes (for a time dt) of the absolute velocities of the charges and $d\mathbf{A}'/dt$ depends on changes of their relative velocities and these changes are <u>equal</u>, and on the changes of the distances between q_i and q which are equal, too.

Putting (23.2) and (23.3) into (8.3), we shall have, remembering the deduction of formula (7.11),

$$\frac{d}{dt}\frac{m(\mathbf{v}+\mathbf{V})}{\{1-(\mathbf{v}+\mathbf{V})^2/c^2\}^{1/2}} = -q(\text{grad}\Phi + \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}) + \frac{q}{c}\,\mathbf{v}\times\text{rot}\mathbf{A} - \frac{q}{c}\,\mathbf{v}\text{div}\mathbf{A} +$$

$$\frac{q\mathbf{v}.\mathbf{V}}{c^2}\,\text{grad}\Phi + \frac{qV^2}{c^2}\,\text{grad}\Phi + \frac{q}{c}\,\mathbf{V}\times\text{rot}\mathbf{A} + \frac{q}{c}\,(\mathbf{V}.\text{grad})\mathbf{A}, \qquad (23.4)$$

where all laboratory quantities in (23.4) and further in this section are written <u>without primes</u>.

Comparing formulas (23.4) and (8.4), we see that their "potential" (right) parts differ with the last four terms in equation (23.4). The electric absolute effects are proportional to V/c and can be neglected with respect to the relative (laboratory) electric effects, however the magnetic absolute effects are not only comparable with the relative magnetic effects but, at V > v, are even bigger.

To demonstrate the validity and effectivity of the relative Newton-Lorentz equation (23.4), let us consider again the rectangular current loop in fig. 5. Let us suppose that the loop moves with a velocity **V** in absolute space and let us attach to it the moving frame K'.

The test charge (the vertical wire in fig. 5) is first at rest in the laboratory, i.e., at rest with respect to the loop, and then it is moved with the laboratory velocity v. The electric intensity induced in the wire as a result of this motion, which can be observed by the help of a voltmeter that is all the time at rest in the laboratory, can be calculated from the following two equations

$$c\mathbf{E} = \mathbf{V}\times\text{rot}\mathbf{A} + (\mathbf{V}.\text{grad})\mathbf{A}, \qquad c\mathbf{E}' = \mathbf{v}\times\text{rot}\mathbf{A} + \mathbf{V}\times\text{rot}\mathbf{A} + (\mathbf{V}.\text{grad})\mathbf{A}, \qquad (23.5)$$

and for the difference E' - E we obtain

$$\mathbf{E}' - \mathbf{E} = \mathbf{E}_{mot} = (\mathbf{v}/c)\times\text{rot}\mathbf{A}. \qquad (23.6)$$

Let us now suppose that the test charge (the vertical wire in fig. 5) is always at rest in the laboratory and the loop originating the magnetic potential first is at rest in the laboratory and then is moved with velocity **v**. The electric intensity induced in the wire as a result of this motion cannot be observed by the help of a voltmeter but only by observing the change of the charges at the extremities of the vertical wire in fig. 5 and can be calculated as follows: The initial induced electric intensity **E** will be the same as in (23.5). When the loop is set in motion with velocity **v**, we have to write the relative Newton-Lorentz equation in a frame K" moving with a velocity **V** + **v** in absolute space, as only in this frame the originated laboratory magnetic potential will be as at the initial moment. As in this frame the test charge will have a velocity - **v**, we obtain

$$c\mathbf{E}'' = - \mathbf{v}\times \mathrm{rot}\mathbf{A} + (\mathbf{v} + \mathbf{V})\times\mathrm{rot}\mathbf{A} + (\mathbf{v} + \mathbf{V}).\mathrm{grad}\,\mathbf{A}, \qquad (23.7)$$

and for the difference **E**" - **E** we obtain

$$\mathbf{E}'' - \mathbf{E} = \mathbf{E}_{mot-tr} = (\mathbf{v}.\mathrm{grad})\mathbf{A}/c. \qquad (23.8)$$

That's the whole "secret" of the space-time absoluteness which neither Lorentz and Poincare nor Einstein and *tutti quanti* could grasp. A problem to be solved by children!

If the loop and the test charge (the vertical wire in fig. 5) are first at rest in the laboratory and then move together with velocity **v**, instead of equation (23.7), we have to write

$$c\mathbf{E}''' = (\mathbf{v} + \mathbf{V})\times\mathrm{rot}\mathbf{A} + \{(\mathbf{v} + \mathbf{V}).\mathrm{grad}\}\mathbf{A}, \qquad (23.9)$$

and for the difference **E**"' - **E** we obtain

$$\mathbf{E}''' - \mathbf{E} = \mathbf{E}_{mot} + \mathbf{E}_{mot-tr} = \mathbf{v}\times\mathrm{rot}\mathbf{A}/c + (\mathbf{v}.\mathrm{grad})\mathbf{A}/c. \qquad (23.10)$$

The different effects described by formulas (23.6), (23.8) and (23.10) were observed first by Faraday on his famous disk[16] with closed loops by using sliding contacts and by Kennard[14] with open loops. By transforming Kennard's rotational experiment to an inertial experiment, called by me the quasi-Kennard experiment, I succeeded (see Sect. 45) to measure the Earth's absolute velocity by using the first formula (23.5).

24. WHITTAKER'S AND NICOLAEV'S FORMULAS

24.1. WHITTAKER'S FORMULA.

Let us consider the Newton-Lorentz equation (8.4) and assume $\mathrm{grad}\Phi = 0$, $\partial\mathbf{A}/\partial t = 0$ and that the magnetic potential **A** is generated by a single current element I'd**r**'

$$\mathbf{A} = I'd\mathbf{r}'/cr. \qquad (24.1)$$

Puttung all this in (8.4) and presenting qv as a current element Id**r**, we shall obtain for the kinetic force of the current element Id**r** (or for the potential force with which the current element I'd**r**' acts on the current element Id**r**) the following expression, where **r** points from **dr**' to dr,

$$df = (II'/c^2)\{dr\times rot(dr'/r) - dr\,div(dr'/r)\} = (II'/c^2r^3)\{dr\times(dr'\times r) + dr(dr'.r)\} =$$

$$(II'/c^2r^3)\{(r.dr)dr' - (dr.dr')r + (r.dr')dr\}. \qquad (24.2)$$

I call (24.2) the WHITTAKER FORMULA, as allegedly Whittaker[17] was the first one who has written it on a piece of paper without presenting some motivations. I write Whittaker's formula also in another form in which the places of the different term are exchanged

$$df = (II'/c^2r^3\{(r.dr')dr + (r.dr)dr' - (dr.dr')r\}. \qquad (24.3)$$

The GRASSMANN FORMULA[18], which can be obtained exactly in the same way from the LORENTZ EQUATION, what is equation (8.4) without the last term, is (24.2) without the last term, i.e.,

$$df = (II'/c^2r^3)\{(r.dr)dr' - (dr.dr')r\}. \qquad (24.4)$$

The AMPERE FORMULA[19] has the form

$$df = (II'/c^2r^5)\{3(r.dr)(r.dr') - 2(dr.dr')r^2\}r. \qquad (24.5)$$

Ampere's formula (24.5) shows that the potential forces with which two current elements act one on another are equal, oppositely directed, and lie on the line joining the two elements. Thus Ampere's formula preserves Newton's third law (at the deduction of his formula Ampere assumed that Newton's third law must be valid at the interaction of two current elements).

Whittaker's formula (24.3) shows that the potential forces with which two current elements act one on another are equal, oppositely directed, but may not lie on the line joining the elements. Thus Whittaker's formula violates Newton's third law.

Grassmann's formula (24.4) shows that the potential forces with which two current elements act one on another may be neither equal nor oppositely directed. This formula drastically violates Newton's third law and all professors in the world are caught by a panic fear when they have to teach it to the students. For this reason, although being the fundamental formula in official magnetism, it can be seen in only one of ten textbooks.

For the force with which a closed current loop L' acts on another closed current loop L all three formulas lead to the same result

$$f = - (II'/c^2)\oint_L\oint_{L'}(dr.dr'/r^3)r, \qquad (24.6)$$

which preserves Newton's third law. The integration of formula (24.3) can easily be carried out as $r.dr/r^3 = - d(1/r)$ and $r.dr'/r^3 = d(1/r)$ are total differentials and at the integratiin along the closed loops L and L', respectively, give zeros.

On the same grounds one sees that Grassmann's formula also leads to formula (24.6).

The conclusion that Ampere's formula also leads to formula (24.6) is based on a theorem demonstrated by Lyness[20] that the force with which a closed current loop acts on a current element is the same according to Ampere's and Grassmann's formulas.

Let me emphasize that according to formula (24.6) the forces with which two current loops act one on another are equal and oppositely directed. Thus for an isolated system consisting of two current loops the momentum conservation law will be conserved. However formula (24.6) does not say whether the torques with which two current loops act one on another will be equal and oppositely directed, thus it does not say whether for an isolated system consisting of two current loops also the angular momentum conservation law will be conserved.

I could not prove this second theorem and to the best of my knowledge there is no such a theorem in the literature (of course when proceeding from Grassmann's formula, as Whittaker's formula is practically unknown).

This aspect for the interaction of the closed current loops remains for me open. As the reader will see in Sects. 50 and 56, I tried to construct machines which had to violate the angular momentum conservation law at the interaction of closed loops but without success and my intuition says that at the interaction of closed loops the angular momentum conservation law cannot be violated.

As shown in Sect. 63, I succeeded to violate the angular momentum conservation law only by constructing a machine with non-closed current loops.

Both Grassmann's and Ampere's formulas are wrong (see Sect. 26, 57, 58, 63) and Whittaker's formula is to be consdiered as the right one. I shall show, however, in Sect. 24.2 that certain theoretical considerations require the introduction of a certain change in Whittaker's formula which thus obtains a slightly different mathematical form, called by me the NICOLAEV FORMULA. It is Nicolaev's formula which is confirmed by the experiments (see Sects. 57 -60).

For the force with which a closed current loop L' acts on a current element $I\mathbf{dr}$ of the loop L we obtain from (24.3), taking again into account that $\mathbf{r}.d\mathbf{r}'/r^3 = d(1/r)$ is a total differential,

$$\Delta f = (II'/c^2)\oint_{L'} \mathbf{dr}\times\mathrm{rot}(d\mathbf{r}'/r) = (Id\mathbf{r}/c)\times\oint_{L'} \mathrm{rot}(I'\mathbf{dr}'/cr) = (Id\mathbf{r}/c)\times\mathbf{B}. \quad (24.7)$$

Thus the Whittaker scalar magnetic intensity produced by a closed current loop is zero. For this reason during two centuries of experimental work humanity could not reveal the existence of the scalar magnetic field.

However, as it will be shown in Sect. 24.2, the Nicolaev scalar magnetic intensity produced by a closed current loop may not be zero and one has to wonder that after two centuries of experimental work Nicolaev was, as a matter of fact, the first one who has observed it in childishly simple experiments.

Before presenting Nicolaev's formula, let me show that if the current elements $Id\mathbf{r}$ and $I'\mathbf{dr}'$ are coplanar, then their Whittaker forces of interaction depend only on the distance between the elements but not on the angles defining their mutual positions. Indeed, according to formula (24.3), omitting the factor (II'/c^2r^2) and denoting by $\mathbf{n} = \mathbf{r}/r$ the unit vector pointing from $\mathbf{dr}'$ to $d\mathbf{r}$, we shall have for the square of the magnitude of the force df with which $I'\mathbf{dr}'$ acts on $Id\mathbf{r}$, taking into taking into account that the angle between **n** and $d\mathbf{r}\times d\mathbf{r}'$ is equal to $\pi/2$.

$$(df)^2 = \{(\mathbf{n}.d\mathbf{r}')d\mathbf{r} + (\mathbf{n}.d\mathbf{r})d\mathbf{r}' - (d\mathbf{r}.d\mathbf{r}')\mathbf{n}\}^2 = \{(\mathbf{n}.d\mathbf{r}')d\mathbf{r} - (\mathbf{n}.d\mathbf{r})d\mathbf{r}'\}^2 + (d\mathbf{r}.d\mathbf{r}')^2 =$$

$$\{\mathbf{n}\times(d\mathbf{r}\times d\mathbf{r}')\}^2 + (d\mathbf{r}.d\mathbf{r}')^2 = dr^2dr'^2\sin^2\alpha + dr^2dr'^2\cos^2\alpha = dr^2dr'^2, \quad (24.8)$$

where α is the angle between $d\mathbf{r}$ and $d\mathbf{r}'$.

24.2. NICOLAEV'S FORMULA.

Let us consider two parallel current elements $Id\mathbf{r}$ and $I'd\mathbf{r}'$ lying on the y-axis and pointing in parallel to the x-axis whose radius vectors are, respectively, 0 and $y\hat{y}$, where $\mathbf{r} = -y\hat{y}$ is the vector distance pointing from the current element $d\mathbf{r}'$ to the current element $d\mathbf{r}$. The force with which $I'd\mathbf{r}'$ acts on $Id\mathbf{r}$, according to Whittaker's formula (24.3), will be

$$df = -(II'/c^2r^3)drdr'\mathbf{r} = (II'drdr'/c^2y^2)\hat{y} \quad (24.9)$$

and will point towards $d\mathbf{r}'$, thus $Id\mathbf{r}$ will be attracted by $I'd\mathbf{r}'$. The current element $Id\mathbf{r}$ will act on the current element $I'd\mathbf{r}'$ with the same and oppositely directed attractive force.

At the mutual attraction of $Id\mathbf{r}$ and $I'd\mathbf{r}'$, their magnetic energy, which is a negative quantity, will decrease (its absolute value will increase) and the loss of magnetic energy will be equal to the gain of mechanical energy, as the kinetic energies of the elements will increase.

Let us now suppose that the same current elements lie on the x-axis pointing again along the x-axis and their radius vectors are, respectively, 0 and $x\hat{x}$, where $\mathbf{r} = -x\hat{x}$ is the vector distance pointing from $d\mathbf{r}'$ to $d\mathbf{r}$. The force with which $I'd\mathbf{r}'$ acts on $Id\mathbf{r}$, according to Whittaker's formula (24.3), will be

$$df = (II'/c^2r^3)drdr'\mathbf{r} = -(II'drdr'/c^2x^2)\hat{x} \quad (24.10)$$

and will point towards $d\mathbf{r}$, thus $Id\mathbf{r}$ will be repulsed by $I'd\mathbf{r}'$. The current element $Id\mathbf{r}$ will act on the current element $I'd\mathbf{r}'$ with the same and oppositely directed repulsive force.

At the mutual repulsion of $Id\mathbf{r}$ and $I'd\mathbf{r}'$, their magnetic energy, which is a negative quantity, will increase (its absolute value will decrease) , but, on the other hand, also the kinetic energies of the two current elements, due to their repulsive forces, will increase. This is a patent violation of the energy conservation law. Thus something is wrong with Whittaker's formula.

There is also another delicate point. We cannot imagine how current elements may move along the current wire. If we have an elastic wire which we can extand mechanically, there will be motion of the line elements, but from an electromagnetic point of view, at such an extension, the electromagnetic system remains exactly the same and there is no motion of the current elements.

Proceeding from these speculations, I decided to write Whittaker's term in Whittaker's formula, i.e., the last term in formula (24.2) or the first term in formula

(24.3), in the following form

$$(\mathbf{r}.d\mathbf{r}')(1 - \frac{(dr.dr')^2}{dr^2dr'^2})d\mathbf{r} = (r.dr')\frac{(d\mathbf{r}\times dr')^2}{dr^2dr'^2}\, d\mathbf{r} \quad (24.11)$$

and I assumed *ad hoc* that the right formula describing the interaction between two current elements is not Whittaker's formula (24.3) but the following one

$$df = (II'/c^2r^3)\{(\mathbf{r}.dr')(d\mathbf{r}\times d\mathbf{r}')^2 d\mathbf{r}/dr^2dr'^2 + (\mathbf{r}.d\mathbf{r})d\mathbf{r}' - (d\mathbf{r}.d\mathbf{r}')\mathbf{r}\}. \quad (24.12)$$

Now the Newton-Lorentz equation is to be written not in the form (8.5) but in the following form

$$E_{glob} = - \mathrm{grad}\Phi - \partial\mathbf{A}/c\partial t + (\mathbf{v}/c)\times\mathrm{rot}\mathbf{A} - (\mathbf{v}/c)\{\mathrm{div}\mathbf{v}\int d\mathbf{A}(\mathbf{v}\times d\mathbf{A})^2/v^2dA^2\}, \quad (24.13)$$

where the integral is to be taken over all charges (current elements) any of whom generates the elementary magnetic potential **dA**.

And the scalar magnetic intensity will be presented not in the form (8.6) but in the folowing form

$$S = - \mathrm{div}\mathbf{v}\int d\mathbf{A}(\mathbf{v}\times d\mathbf{A})^2/v^2dA^2, \quad (24.14)$$

i.e., S will depend not only on the electric charges (and their velocities) of the surrounding system and on their distances to the test charge, but also on the direction of motion of the test charge. Thus the scalar magnetic intensity of a given system acting on two test charges with different directions of motion are not equal.

I call formula (24.12) NICOLAEV'S FORMULA and equation (24.13) the NEWTON-LORENTZ EQUATION IN ITS NICOLAEV'S FORM. Equation (8.5) will be then called the NEWTON-LORENTZ EQUATION IN ITS WHITTAKER'S FORM. And now the Whittaker electric intensity (21.1) is to be substituted by the NICOLAEV ELECTRIC INTENSITY

$$E_{nic} = - (\mathbf{v}/c)\mathrm{div}\mathbf{v}\int d\mathbf{A}(\mathbf{v}\times d\mathbf{A})^2/v^2dA^2, \quad (24.15)$$

where the integral is taken over the surrounding system, every current element of which generates the elementary magnetic potential **dA**.

Here I have to note that the equation of potential connection (8.8) preserves its validity, but we can no more replace Nicolaev's equation (24.13) by equation (8.9), so that the calculation of the global electric intensity is to be done proceeding only from Nicolaev's equation (24.13).

The reader has seen in Sect. 7 that the introduction of the Whittaker's term in equation (7.9), i.e., the middle term on the right side of equation (7.9), was not sufficiently lawful from a rigorous mathematical point of view. And now I make another completely *ad hoc* deformation of this formula. Thus the conclusion is to be done that the Divinity, when constructing the theoretical basis of electromagnetism, proceeding from the axiomatical Coulomb, Neumann and Newton laws, and when seeing that the theory leads to some unpleasant contradictions, trampling with both feet on the rigorous mathematical logic, introduced some "hocus pocus" tricks which no earthly scientist would allow himself to do.

What can I do, dear reader? You see, the Divinity is not perfect: *errare divinum est*. And I am only his prophet.

To a certain degree I can accept the introduction of the second term on the right side of equation (7.9) as a correct mathematical path (my friend Prof. U. Bartocci insists that the introduction of this term is inadmissible from a rigorous mathematical point of view). Indeed "physical mathematics" permits certain "frivolities" but the introduction of "Nicolaev's correction" in the Whittaker's term is a complete mathematical fiasco. If Nicolaev's formula is the right one and the Divinity was perfect, He had to arrive at this formula by logical mathematical steps.

When one introduces similar logical acrobatics in the edifice of electromagnetism, one cannot more be sure whether the fundamental axioms will preserve their absolute validity. And if on our Earth there are clever children recognizing the Mephistophelian mathematical manipulations of the Divinity, they will be able to construct machines violating the most divine of all divine laws - the law of energy conservation (see Sect. 60).

I must, of course, declare that I am not sure whether formula (24.12) introduced by me is the right one. The way to establish whether it is the right one is the following: The effects predicted by Nicolaev's formula for all known fundamental experiments are to be calculated on a computer. If always the formula will give the right prediction, it is to be accepted as right until the day when somebody will show that the right formula is another one.

I called formula (24.12) Nicolaev's formula, as the Russian physicist of Tomsk Genadi Nicolaev, whom I met at the space-time conference in Saint Petersburg in 1991, has done many experiments (see Sect. 58) showing that a formula of such a kind must be the right one.

It is possible, of course, that the Divinity has not changed *ad hoc* the Whittaker term into the Nicolaev term. Maybe the Divinity writes the space-time energy of two electric charges q_1, q_2 moving with velocities $\mathbf{v}_1$, $\mathbf{v}_2$ not in the Neumann's form (2.14) but in the following form

$$W = -(q_1 q_2 \mathbf{v}_1 . \mathbf{v}_2 / c^2 r^3)(\mathbf{v}_1 \times \mathbf{r})(\mathbf{v}_2 \times \mathbf{r})/v_1 v_2, \qquad (24.16)$$

or in the form

$$W = -(q_1 q_2 \mathbf{v}_1 . \mathbf{v}_2 / c^2 r^3)\{(\mathbf{v}_1 - \mathbf{v}_2)\times \mathbf{r}\}^2/(\mathbf{v}_1 - \mathbf{v}_2)^2. \qquad (24.17)$$

Now, perhaps, the Divinity will come to Nicolaev's formula on a rigorous mathematical way. I leave to the mathematicians the honour to prove this hypothesis, but I must declare that the form (24.16) is complicated, unesthetic, and if the Divinity is a Divinity He would not choose such a ghastly expression in His axiomatics.

In the next three sections I shall make calculations of the forces acting between the current wires in some simple but fundamental circuits. As pretty many experiments have shown that Grassmann's and Ampere's formulas are wrong (see Chapter VI), the formulas which still remain competitive are the Whittaker and Nicolaev formulas.

Thus the calculation of the forces of interaction between current wires will be done when proceeding from Whittaker's and Nicolaev's formulas. In certain fundamnetal cases only, in order to reveal the differences, calculations also according to Grassmann's and Ampere's formulas will be done.

25. THE PROPULSIVE AMPERE BRIDGE (PAB)

The calculation of the magnetic force with which a closed current loop acts on a current element or on another open or closed loop is a simple calculation problem. However when we have to calculate the magnetic force with which a current loop acts on some of its current elements or a part of a current loop acts on other its part, inconveniences may appear, as the integrals may contain singularities. In such cases we have to make use of certain calculation tricks to be able to evaluate the acting forces.

As a first example, I shall calculate the force with which the current in one half of a circle of radius R and wire's radius r acts on the currenr in the other half. This force can be measured if at the points where the two half-circles make contact sliding contacts will be put.

If we shall try to use Whittaker's formula (24.3) or Nicolaev's formula (24.12), taking as L' the one half of the circle ans as L the other half, we shall obtain an integral containing singularities, so that we must search for another way to solve the problem.

According to formulas (18.20) and (18.9), the magnetic energy of this circle when current I flows in it will be

$$W = - \sqrt{2}\pi^2 I^2 R^{3/2}/c\sqrt{r}. \qquad (25.1)$$

At an increase of the radius with dR, the magnetic energy will increase by dW and the magnitude of the force acting on an element dr_0 of the circuit will be

$$df = (dr_0/2\pi R)(dW/dR) = 3\pi dr_0 I^2/2\sqrt{2}c^2\sqrt{rR}. \qquad (25.2)$$

This force is perpendicular to dr_0 and obviously directed outside of the circle. Thus if the circular wire is done of elastic material, it will expand <u>delivering</u> mechanical energy and <u>decreasing</u> its magnetic energy.

To obtain the net force acting on one half of the circle, we have to write in (25.2) $dr_0 = Rd\phi$ and to take the projection of the force acting on dr_0 along the central radius of the half circle. Taking then into account that in a half circle there are two fourth circles, we shall have for the net force

$$f = 2\int_0^{\pi/2} df\sin\phi = 2\int_0^{\pi/2} \frac{3\pi I^2 R\sin\phi\, d\phi}{2\sqrt{2}c^2\sqrt{rR}} = (3\pi/\sqrt{2}c^2)I^2\sqrt{R/r}. \qquad (25.3)$$

Thus the force pushing any of the two half-circles is proportional to the square root of R/r.

When the one half-circle is fixed to the laboratory and the other has sliding contacts and is free to move, we call it CIRCULAR PROPULSIVE AMPERE BRIDGE. Of course, when the half circle has moved a little, the circuit is no more circular and the pushing force may change.

In fig. 6 the HALF-CIRCULAR PROPULSIVE AMPERE BRIDGE is shown. The half circle is called SHOULDER of the bridge and the vertical wires are called ARMS of the bridge. With the notations given in fig. 6 I have calculated[21] the force pushing the half circle upwards when there are sliding contacts at the tops of the arms by using Whittaker's formula (24.3). The obtained integral which, of course, has singularities is given in Ref. 21. I could not find a way to evaluate the force pushing the half-circular Ampere bridge but it surely must be near (if not equal) to the force (25.3).

The classical half circular PROPULSIVE AMPERE BRIDGE (PAB) experiment was done by Ampere in 1823 and is presented in fig. 7. The difference between the bridges in figs. 6 and 7 is that in the former the bridge is in the plane of the arms, while in the latter it is perpendicular to the plane of the arms. The pushing force acting on these two bridges surely must be the same.

Ampere filled the troughs in fig. 7 with mercury, so that excellent sliding contacts have been realized. Tait exchanged the copper bridge of Ampere by a glass tube filled with mercury to show that the effect is magnetic and not due to some surface forces at the contact mercury-copper.

Instead of the half-circle in figs. 6 and 7 one can put a shoulder with a linear

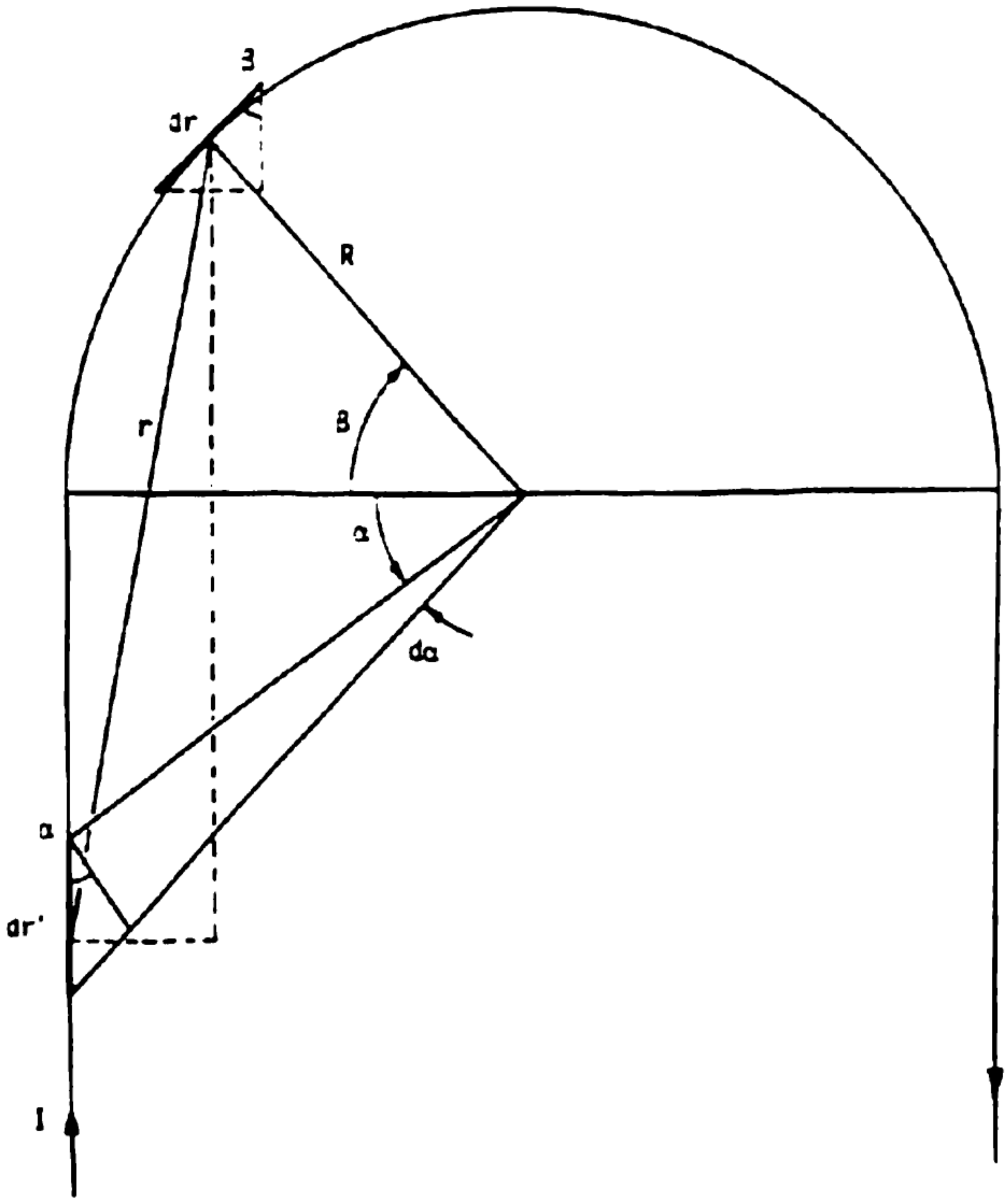

Fig. 6. Half-circular propulsive Ampere bridge.

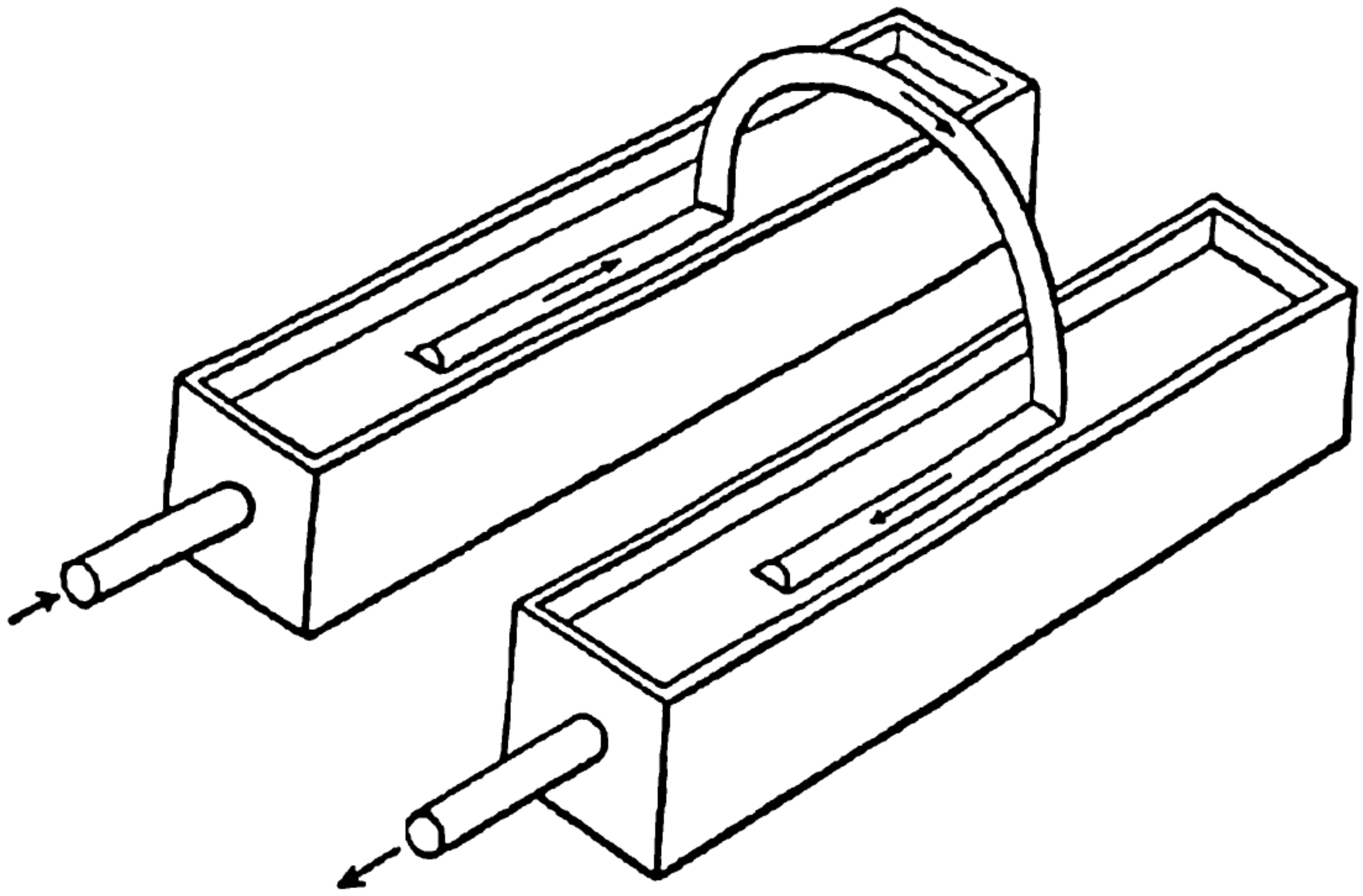

Fig. 7. The classical propulsive Ampere bridge.

form or with a Π-form.

The arms of the Ampere bridge can be done very long (theoretically one can assume them infinitely long) and the sliding contacts can be put at any two points at equal distances from the shoulder, so that the upper parts of the arms will be propulsive and lower stationary.

According to Nicolaev's formula, as there are no forces between colinear currents, with the increase of the propulsive arms the pushing force in the half-circular Ampere bridge must diminish. As far as I know, measurements for establishing the existence (or non-existence) of such an effect have not been done.

On the other hand, the change in the magnetic energy of the whole circuit of the Ampere bridge does not depend on the fact at which points of the arms the sliding contacts are taken and thus, for a definite circuit, the pushing force cannot depend on the relation between the propulsive and stationary arms. Here one has to take also into account that when increasing the length of the propulsive arms a pushing force acting on these propulsive arms appears generated by the current in the "opposite" shoulder.

26. ACTION OF RECTANGULAR CURRENT ON A PART OF IT

26.1. CALCULATION WITH WHITTAKER'S FORMULA.

Now I shall calculate the longitudinal magnetic force acting on the current wire BC in the rectangular circuit ODEF in fig. 8. It was claimed by Nicolaev[21] that there is a longitudinal force acting on the wire BC and that he has observed it. Now I shall show that, according to Whittaker's formula the net longitudinal force acting on the current BC is null.

The wire BC can slide at the contacts B and S and has the length L. The action of the currents between points A and B and between points C and D on the current in the wire BC is entirely symmetric and opposite, so that the force acting on BC will be determined by the action of the currents in the wires OA, with length D, DE and FO, with lengths H, and EF, with length $D+L+2a$.

First, for more simple calculation, I shall assume that D and H are <u>very long</u>, so that the action of the currents EF and FO can be neglected. Whittaker's formula (24.3) gives for the x-component of the force (equal to the total force) with which the current OA acts on the current BC, by denoting $dr = dx$, $dr' = dx'$, $r = x+a+x'$, where $x' = 0$ at point A and $x = 0$ at point B (the last two assumptions lead to more simple limits in the integrals),

$$(f_{OA})_x = (I^2/c^2)\int_B^C \int_0^A dr dr'/r^2 = (I^2/c^2)\int_0^L dx \int_0^\infty dx'/(x'+a+x)^2 = (I^2/c^2)\int_0^L dx/(x+a) =$$

$$(I^2/c^2)\ln(1+L/a). \qquad (26.1)$$

For the x-component of the force with which the current DE acts on the current BC we obtain, denoting $dr = dx$, $dr' = dy$, $r = \{(x+a)^2 + y^2\}^{1/2}$ and taking $x = 0$ at point C

$$(f_{DE})_x = (I^2/c^2)\int_B^C \int_D^E (\mathbf{r}.d\mathbf{r}')dr/r^3 = -(I^2/c^2)\int_0^L dx \int_0 y dy/\{(x+a)^2+y^2\}^{3/2} =$$

$$-(I^2/c^2)\int_0^L dx/(x+a) = -(I^2/c^2)\ln(1+L/a). \qquad (26.2)$$

Comparing formulas (26.1) and (26.2), we see that according to Whittaker's formula there is no force acting on the wire BC.

Formulas (26.1) and (26.2) show that, if $x' = y$, the current elements along the longitudinal wire OA which are near to point A act on the current elements along the wire BC with larger forces than the current elements along the transverse wire DE which are near to point D (put, for example, $x' = y = 0$). When the distances $x' = y$ become larger and larger the first forces diminish more rapidly than the second forces, for certain $x_0' = y_0 = b$ they become equal and then the first forces become less than the second ones. By equalizing the elementary forces in (26.1) and (26.2) and by putting there $x_0' = y_0 = b$, we obtain

$$1/(b+a+x)^2 = b/\{(x+a)^2 + b^2\}^{3/2}, \qquad (26.3)$$

from where we can find b as a function of a and x.

Let us now find the net longitudinal force acting on the current BC when the action of the currents in EF and FO cannot be neglected. The integration will be more complicated but in the same lines as in the above two formulas; remembering that

$$\int(1+x^2)^{-1/2}dx = \text{Arsinh}\,x = \ln\{x + (1+x^2)^{1/2}\}, \qquad (26.4)$$

we shall have:

The x-component of the force with which the current OA acts on the current BC will be

$$(f_{OA})_x = (I^2/c^2)\int_0^L dx \int_0^D dx'/(x'+a+x)^2 = (I^2/c^2)\ln\frac{(D+a)(L+a)}{a(D+L+a)}. \qquad (26.5)$$

The x-component of the force with which the current DE acts on the current BC will be

$$(f_{DE})_x = (I^2/c^2)\int_0^L dx \int_0^H y\,dy/\{(x+a)^2 + y^2\}^{3/2} = (I^2/c^2)\ln\frac{a[L + a + \{H^2 + (L+a)^2\}^{1/2}]}{(L+a)\{a + (H^2 + a^2)^{1/2}\}} \quad (26.6)$$

The x-component of the force with which the current FO acts on the current BC can be found directly from the result (26.6) taking it with negative sign and exchanging a for D+a

$$(f_{FO})_x = (I^2/c^2)\ln\frac{(D+L+a)[D + a + \{H^2 + (D+a)^2\}^{1/2}]}{(D+a)[D + L + a + \{H^2 + (D+L+a)^2\}^{1/2}]}. \quad (26.7)$$

The x-component of the force with which the current EF acts on the current BC will be, if taking x' = 0 at point F and dividing the integral on x' into two integrals, as for x' < D+a+x the x-component of the force is negative and for x' > D+a+x positive,

$$(f_{EF})_x = -\frac{I^2}{c^2}\int_0^L dx \int_0^{D+a+x} \frac{(D+a+x-x')dx'}{\{(D+a+x-x')^2 + H^2\}^{3/2}} + \frac{I^2}{c^2}\int_0^L dx \int_{D+a+x}^{D+L+2a} \frac{(x'-D-a-x)dx'}{\{(x'-D-a-x)^2 + H^2\}^{3/2}} =$$

$$(I^2/c^2)\ln\frac{\{a + (H^2 + a^2)^{1/2}\}[D + L + a + \{H^2 + (D+L+a)^2\}^{1/2}]}{[D + a + \{H^2 + (D+a)^2\}^{1/2}][L + a + \{H^2 + (L+a)^2\}^{1/2}]}. \quad (26.8)$$

The net longitudinal force acting on the wire BC will be the sum of the forces

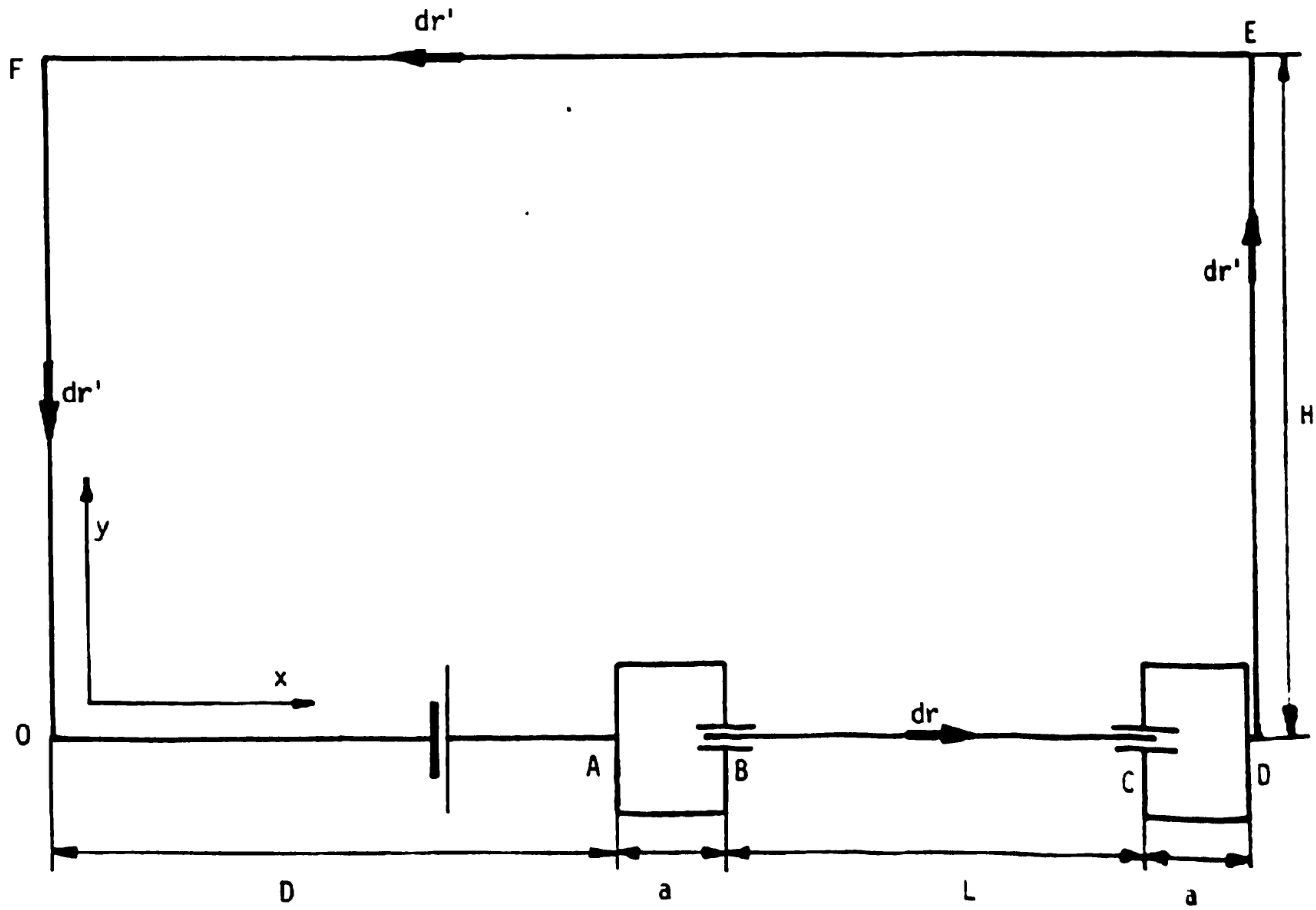

Fig. 8. Rectangular current loop acting on a part of it.