(26.5), (26.6), (26.7) and (26.8) and it is equal to zero

$$(f_{OA})_x + (f_{DE})_x + (f_{FO})_x + (f_{EF})_x = (I^2/c^2)\ln 1 = 0. \qquad (26.9)$$

26.2 CALCULATION WITH NICOLAEV'S FORMULA.

To obtain the prediction of Nicolaev's formula for the force with which the current in the open loop DEFOA acts on the current in the straight wire BC, at the assumption that the wires OA and DE are very long, we have to put in (26.1) $(\mathbf{f}_{OA})_x = 0$ and the force which will remain to act on the wire BC will be only the force $(\mathbf{f}_{DE})_x$ given by formula (26.2). Thus the wire BC will move to the left, as Nicolaev first has observed (see Sect. 58.4). I repeated Nicolaev's experiment in a very impressive variation where a continuous rotation could be observed (see Sect. 59).

26.3. CALCULATION WITH GRASSMANN'S FORMULA.

As according to Grassmann's formula (24.4) the forces acting on a current element must be always perpendicular to the latter, no longitudinal force can act on the current wire BC.

26.4. CALCULATION WITH AMPERE'S FORMULA.

Here also as above the force acting on BC will be determined by the action of the currents in the wires OA and DE. Ampere's formula (24.5) gives for the x-component of the force (equal to the total force) with which the current in OA acts on the current in BC, by denoting dr = dx, dr' = dx', r = x' + x,

$$(f_{OA})_x = (I^2/c^2)\int_B^C\int_O^A dr dr'/r^2 = (I^2/c^2)\int_o^L dx \int_o^\infty dx'/(x' + a + x)^2 = (I^2/c^2)\int_o^L dx/(x+a) = (I^2/c^2)\ln(1 + L/a). \qquad (26.10)$$

The forces with which the current elements along the wire DE act on the current elements along the wire BC are directed along the vector distance **r**. We have to consider only the components parallel to BC. The x-component of the force $\mathbf{df}_{DE}$ with which the current element I'dr' along the wire DE acts on the current element I**dr** along the wire BC will be obtained by multiplying $\mathbf{df}_{DE}$ by -**dr**/dr, and denoting dr = dx, dr' = dy, $r = \{(x+a)^2 + y^2\}^{1/2}$, so that for the net force we obtain

$$(f_{DE})_x = \frac{I^2}{c^2}\int_B^C\int_D^E \frac{3(\mathbf{r}.\mathbf{dr})(\mathbf{r}.\mathbf{dr}')}{r^5}\,\frac{\mathbf{r}.(-\mathbf{dr})}{dr} = -(I^2/c^2)\int_o^L 3(x+a)^2 dx \int_o^\infty \frac{y\,dy}{\{(x+a)^2 + y^2\}^{5/2}} =$$

$$-(I^2/c^2)\int_o^L dx/(x+a) = -(I^2/c^2)\ln(1 + L/a). \qquad (26.11)$$

Comparing formulas (26.10) and (26.11), we see that according to Ampere's formula there is no force acting on the wire BC.

Thus the only formula which predicts motion of the wire BC in the rectangular loop ODEF remains Nicolaev's formula.

27. INTERACTION BETWEEN CIRCULAR, RADIAL AND AXIAL CURRENTS

It is highly important to know the forces of interaction between a circular current, on one side, and radial and axial currents, on the other side. To the best of my knowledge, nobody has calculated these forces, even with the wrong Grassmann and Ampere formulas.

Let us consider the most simple circuit consisting of a circular current with radius R and a rectangular current acde perpendicular to it with its corner at the center of the circular current (fig. 9). This case is presented also in fig. 10 where two sliding contacts are put, so that one can observe the appearing forces, as done by Sigalov[21] (I call the experiment shown in fig. 10 the FIRST SIGALOV'S EXPERIMENT). In the single circuit of fig. 10 the current is I, in the two circuits of fig. 9 the currents can be different, I and I'.

I shall calculate the torques (moment of forces) about the axis ac (the z-axis) appearing because of the action:

1. of the internal radial current on the circular current,
2. of the circular current on the internal radial current,
3. of the external radial current on the circular current,
4. of the circular current on the external radial current,
5. of the axial current on the circular current.

As in fig. 9 there are no colinear current elements, both Whittaker's and Nicolaev's formulas will lead to the same or to very similar results. I shall make all calculations in this section according to Whittaker's formula.

For brevity, in all formulas of this section the factor II'/c^2 will be omitted.

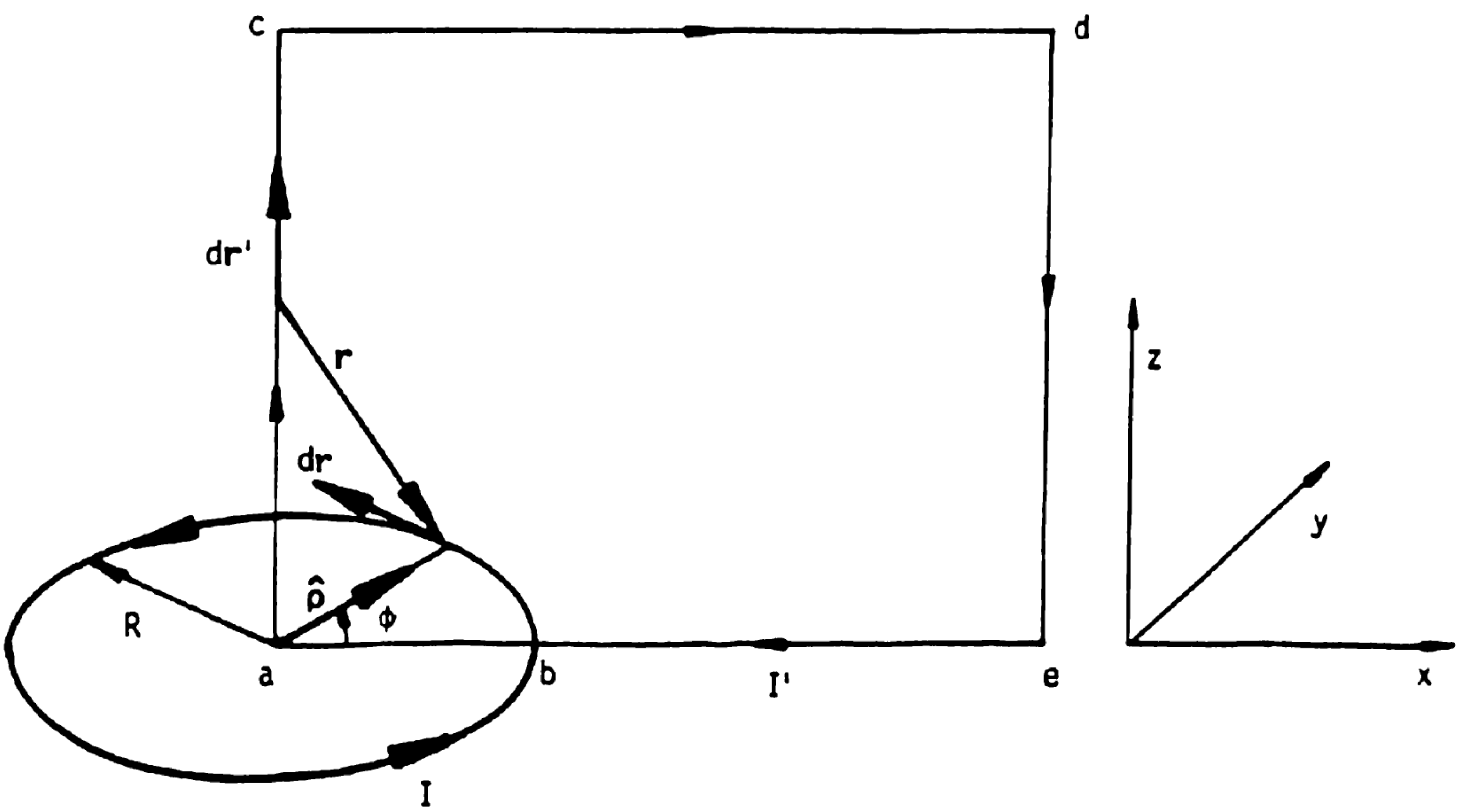

Fig. 9. Rectangular and circular circuits.

27.1. ACTION OF THE INTERNAL RADIAL CURRENT ON THE CIRCULAR CURRENT.

The unit vector along the x-axis is denoted by $\hat{x}$, the unit vector along the polar radius is denoted by $\hat{\rho}$, the unit vector which is perpendicular to the polar radius and corresponds to the polar angle ϕ is denoted by $\hat{\phi}$, and the unit vector along the z-axis is denoted by $\hat{z}$. The circular and internal radial currents are shown in fig. 11.

The elementary moment of force about the z-axis appearing as a result of the action of the radial current element $\mathbf{dr}'$ on the circular current element dr will be

$$\mathbf{dM} = R\hat{\rho}\times d\mathbf{f}, \tag{27.1}$$

so that by substituting (24.3) into (27.1) we obtain

$$dM = (R/r^2)\hat{\rho}\times\{\cos\psi(-\hat{x}) + \cos\psi'\hat{\phi} - \sin\phi(\mathbf{r}/r)\}drdr'. \tag{27.2}$$

As

$$\mathbf{r}/r = \sin\psi\hat{\rho} + \cos\psi\hat{\phi}, \quad dr = Rd\phi, \quad dr' = dx, \quad \hat{\rho}\times\hat{x} = -\sin\phi\hat{z}, \quad \hat{\rho}\times\hat{\phi} = \hat{z}, \tag{27.3}$$

we obtain

$$\mathbf{dM} = (R^2/r^2)\cos\psi' dx\, d\phi\hat{z}. \tag{27.4}$$

We have from fig. 11

$$\sin\psi' = R\sin\phi/r, \qquad r^2 = x^2 - 2xR\cos\phi + R^2, \tag{27.5}$$

so that by putting (27.5) into (27.4) we obtain for the component of the elementary torque about the z-axis

$$dM = \frac{R^2}{r^2}\left(1 - \frac{R^2\sin^2\phi}{r^2}\right)^{1/2} dx d\phi = \frac{R^2(x - R\cos\phi)\, dx d\phi}{(x^2 - 2xR\cos\phi + R^2)^{3/2}}. \tag{27.6}$$

For $x > R\cos\phi$ the torque is <u>positive</u> and for $x < R\cos\phi$ <u>negative</u> (see fig. 11).

To obtain the torque M acting on the whole circular current, we have to integrate formula (27.6) for x in the limits from 0 to R and for ϕ in the limits from 0 to 2π.

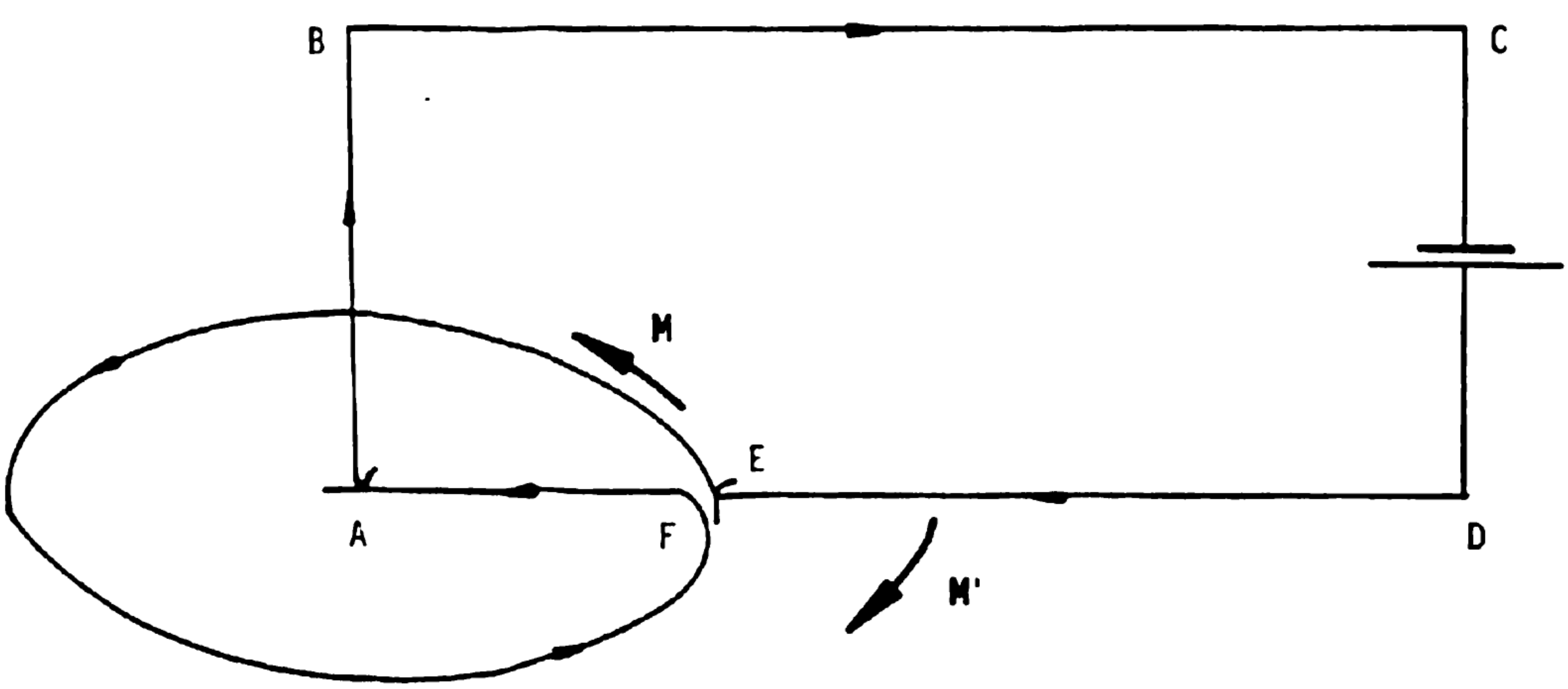

Fig. 10. Sigalov's first experiment.

Both integrations can easily be carried out in a final form, however at the point $x = R$, $\phi = 0$ there is a singularity: the distance between $\mathbf{dr}$ and $\mathbf{dr}'$ becomes equal to zero. Thus we shall write the solution in the following form

$$M = \int_0^{2\pi} d\phi \int_0^R \frac{R^2(x - R\cos\phi)dx}{(x^2 - 2xR\cos\phi + R^2)^{3/2}} = - \int_0^{2\pi} \frac{Rd\phi}{2\sin(\phi/2)} + \int_0^{2\pi} Rd\phi = - R\ln\frac{\tan\pi}{\tan 0} + 2\pi R. \quad (27.7)$$

27.2. ACTION OF THE CIRCULAR CURRENT ON THE INTERNAL RADIAL CURRENT.

To find the torque with which the circular current acts on the radial current, we change the directions of the currents I and I' to the opposite. In such a case the acting forces remain the same, but we shall have now the angles ψ and ψ' less than $\pi/2$ and this will facilitate the matemantics (fig. 12).

The elementary torque about the z-axis appearing as a result of the action of the circular current element $\mathbf{dr}'$ on the radial current element $\mathbf{dr}$ will be

$$\mathbf{dM} = x\hat{\mathbf{x}}\times\mathbf{df}, \quad (27.8)$$

so that by substituting (27.2) into (27.8) we obtain

$$\mathbf{dM} = (x/r^2)\hat{\mathbf{x}}\times\{\cos\psi(-\hat{\boldsymbol{\phi}}) + \cos\psi'\hat{\mathbf{x}} - \sin\phi(\mathbf{r}/r)\}drdr'. \quad (27.9)$$

As $\mathbf{r}/r = -\sin\psi'\hat{\boldsymbol{\rho}} - \cos\psi'\hat{\boldsymbol{\phi}}$, $dr = dx$, $dr' = Rd\phi$, $\hat{\mathbf{x}}\times\hat{\boldsymbol{\phi}} = \cos\phi\hat{\mathbf{z}}$, $\hat{\mathbf{x}}\times\hat{\boldsymbol{\rho}} = \sin\phi\hat{\mathbf{z}}$, we obtain

$$\mathbf{dM} = (x/r^2)\{-\cos\phi\cos\psi + \sin\phi(\sin\phi\sin\psi' + \cos\phi\cos\psi')\hat{\mathbf{z}} =$$
$$(x/r^2)(-\cos\phi\cos\psi + \sin\phi\sin\psi)\hat{\mathbf{z}}, \quad (27.10)$$

as $\phi - \psi' = \pi/2 - \psi$.

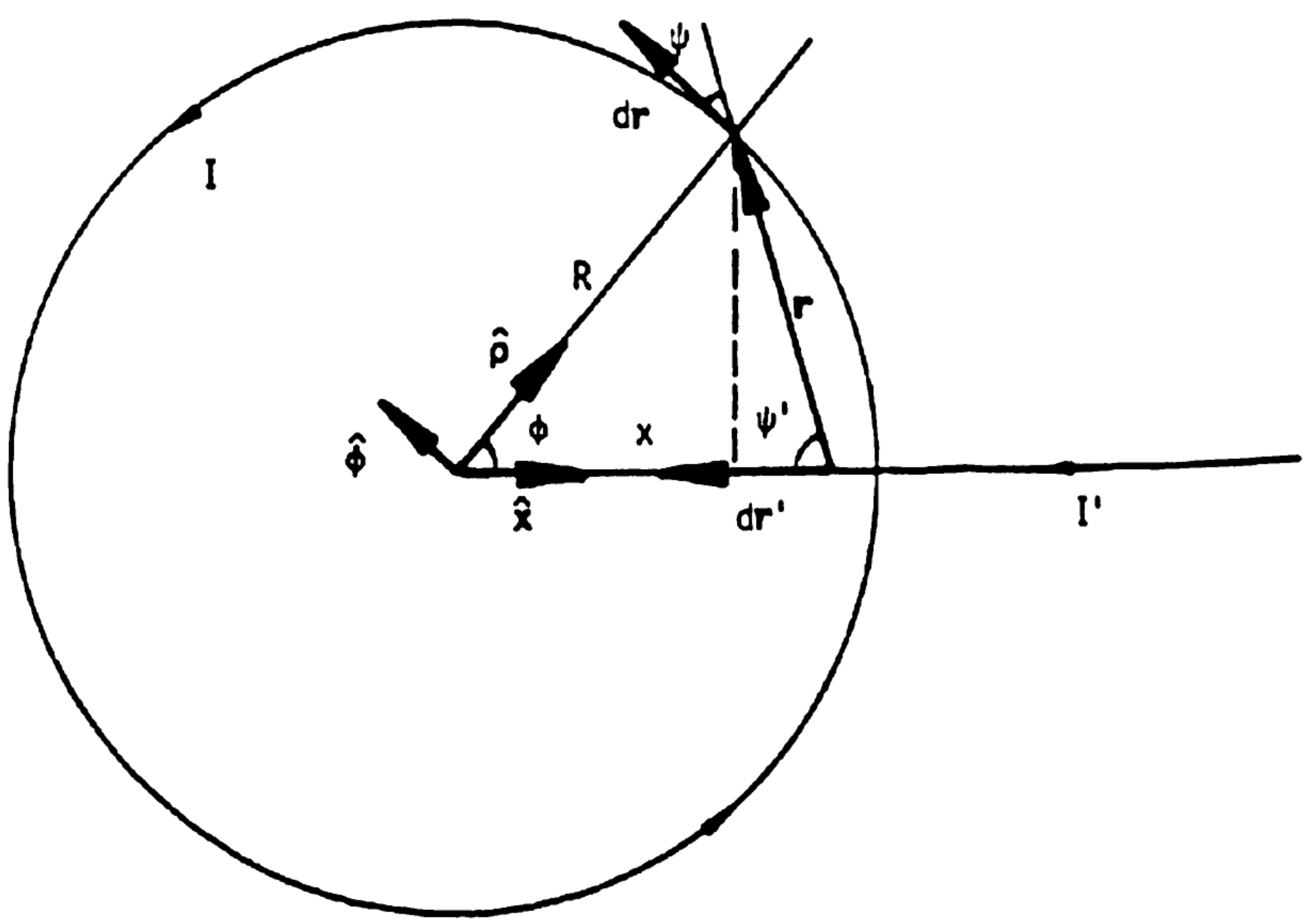

Fig. 11. Action of internal radial current on circular current.

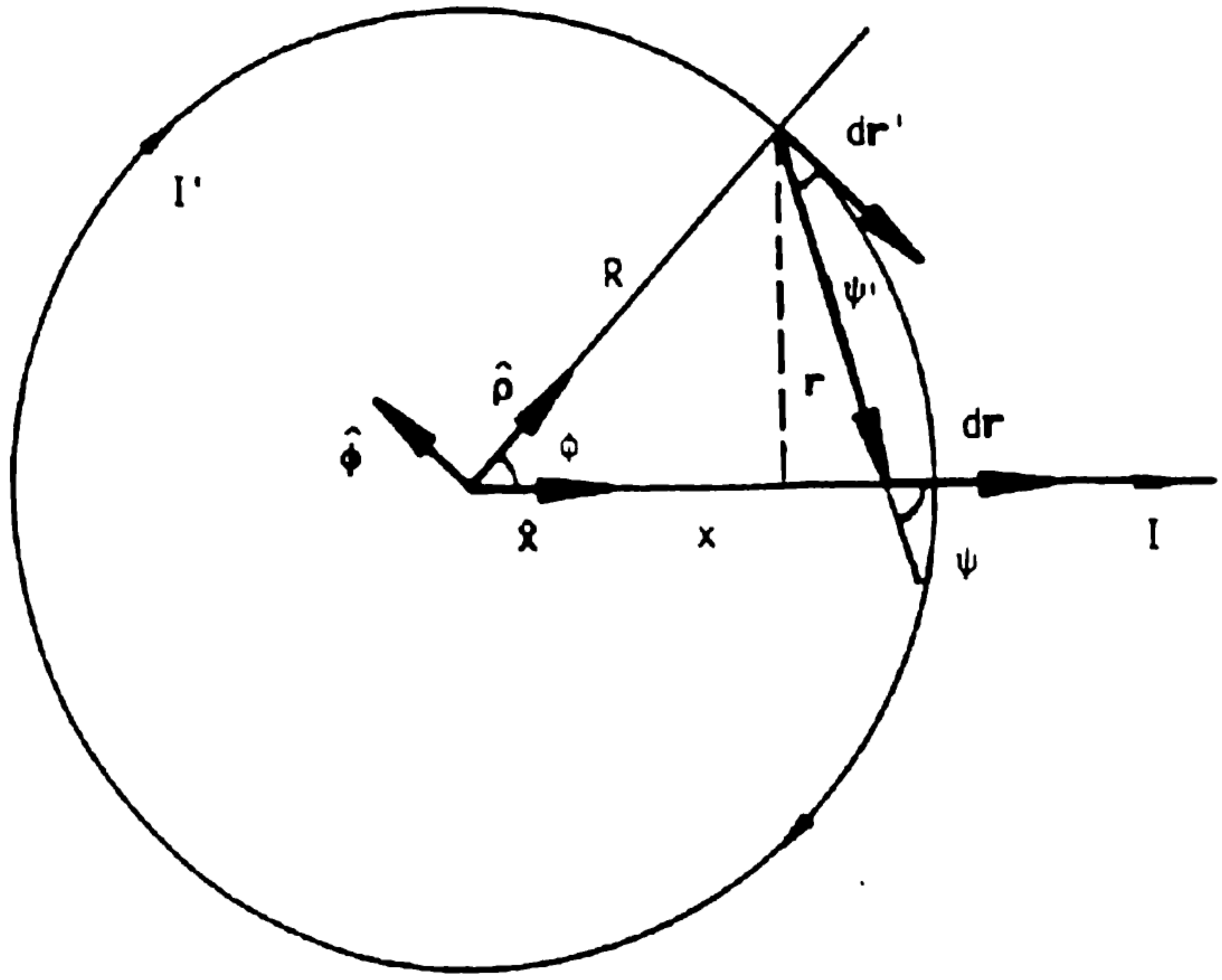

Fig. 12. Action of circular current on internal radial current.

We have from fig. 12

$$\cos\psi = (x - R\cos\phi)/r, \qquad \sin\psi = R\sin\psi/r, \qquad r^2 = R^2 - 2xR\cos\phi + x^2, \qquad (27.11)$$

so that by putting (27.11) into (27.10) we obtain for the z-component of the torque

$$dM = (xR/r^3)\{\cos\phi(R\cos\phi - x) + R\sin^2\phi\}dxd\phi = \frac{xR(R - x\cos\phi)\ dx\,d\phi}{(R^2 - 2xR\cos\phi + x^2)^{3/2}}. \qquad (27.12)$$

As $R > x\cos\phi$, the torque is directed along the z-axis and this leads to anti-clockwise rotation.

To obtain the torque acting on the whole internal radial current, we have to integrate formula (27.12) for x in the limits from 0 to R and for ϕ in the limits from 0 to 2π. I could not evaluate the integral in elementary functions and perhaps this is not possible (the mathematicians have the last word). As the integral for $x = R$, $\phi = 0$, has singularity, I shall write it as a positive number B which is infinitely large

$$M = \int_0^{2\pi} d\phi \int_0^R \frac{xR(R - x\cos\phi)\ dx\,d\phi}{(R^2 - 2xR\cos\phi + x^2)^{3/2}} = B. \qquad (27.13)$$

27.3. ACTION OF THE EXTERNAL RADIAL CURRENT ON THE CIRCULAR CURRENT.

The elementary torque about the z-axis appearing as a result of the action of the external radial current element dr' on the circular current element **dr** will be given by formula (27.1), so that by substituting (24.3) into (27.1) we obtain (fig. 13)

$$dM = (R/r^2)\hat{\rho}\times\{\cos\psi(-\hat{x}) + \cos\psi'\hat{\phi} - \sin\phi(\mathbf{r}/r)\}. \qquad (27.14)$$

As $\mathbf{r}/r = \sin\psi\hat{\rho} + \cos\psi\hat{\phi}$, $dr = Rd\phi$, $dr' = dx$, $\hat{\rho}\times\hat{x} = -\sin\phi\hat{z}$, $\hat{\rho}\times\hat{\phi} = \hat{z}$, we obtain

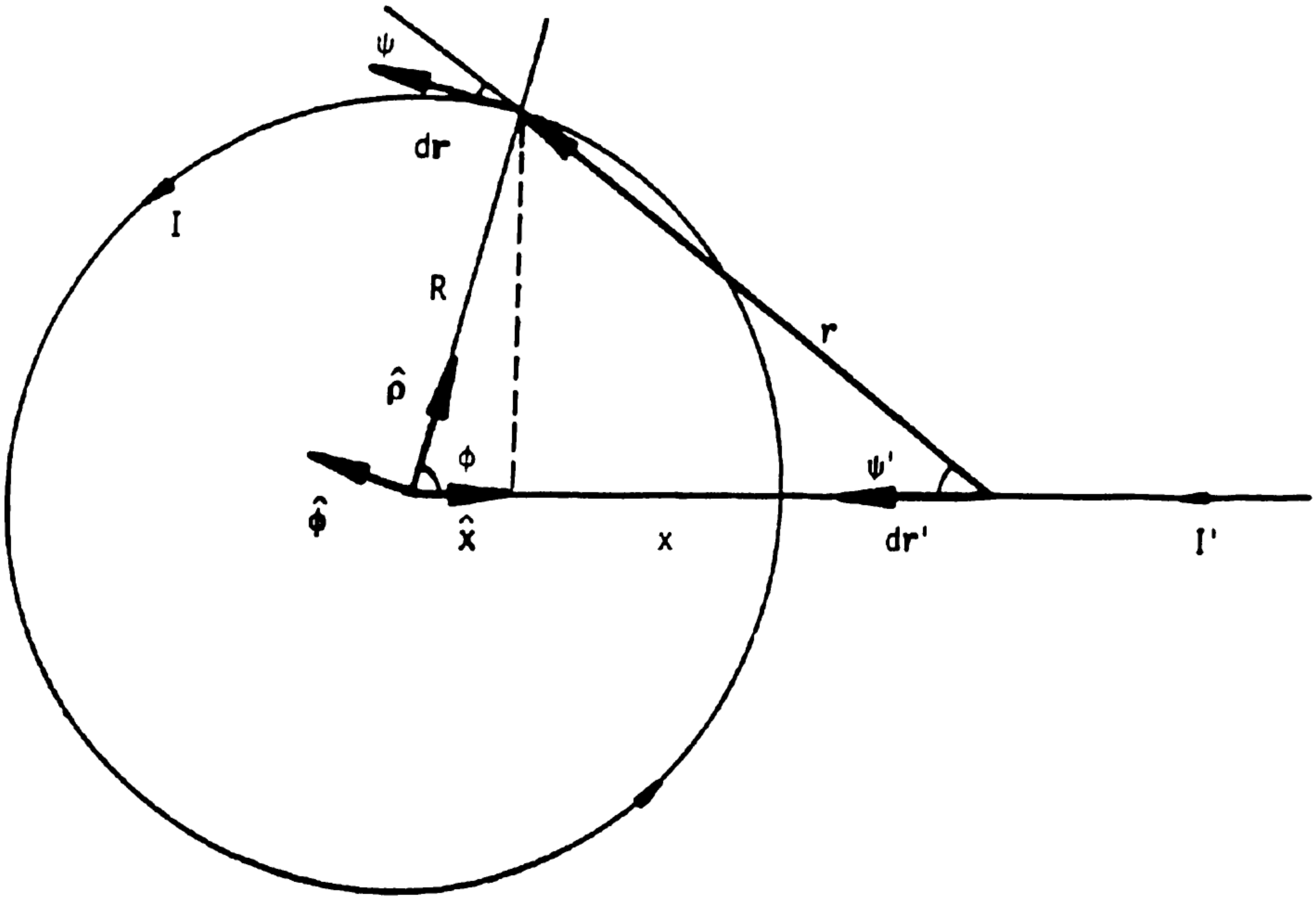

Fig. 13. Action of external radial current on circular current.

$$dM = (R^2/r^2)\cos\psi' dx d\phi \hat{z}. \tag{27.15}$$

We have from fig. 13

$$\sin\psi' = R\sin\phi/r, \qquad r^2 = x^2 - 2xR\cos\phi + R^2, \tag{27.16}$$

so that by putting (27.16) into (27.15) we obtain for the z-component of the elementary torque

$$dM = \frac{R^2}{r^2}\left(1 - \frac{R^2\sin^2\phi}{r^2}\right)^{1/2} dx d\phi = \frac{R^2(x - R\cos\phi)\, dx d\phi}{(x^2 - 2xR\cos\phi + R^2)^{3/2}}. \tag{27.17}$$

As $x > R\cos\phi$, the torque is directed along the z-axis and thus leads to anti-clockwise rotation.

To obtain the torque acting on the whole circular current, we have to integrate formula (27.17) and we obtain

$$M = \int_0^{2\pi} d\phi \int_R^{\infty} \frac{R^2(x - R\cos\phi)dx}{(x^2 - 2xR\cos\phi + R^2)^{3/2}} = \int_0^{2\pi} \frac{R d\phi}{2\sin(\phi/2)} = R\ln\frac{\tan\pi}{\tan 0}. \tag{27.18}$$

Taking into account formulas (27.7) and (27.18), we see that the torque which the internal and external radial currents exerts on the circular current is finite and equal to $2\pi R$.

27.4. ACTION OF THE CIRCULAR CURRENT ON THE EXTERNAL RADIAL CURRENT.

Here again as in Sect. 27.2 we exchange the directions of the circular and radial currents to the opposite to have the angles ψ and ψ' less than $\pi/2$.

The elementary torque about the z-axis appearing as a result of the action of the circular current element dr' on the radial current element dr can be obtained exactly in the same way as in Sect. 27.2 (fig. 12 can be used by considering the element dr outside the circle). For the z-component of the acting elementary torque we shall obtain formula (27.12).

For $R > x\cos\phi$ the torque is positive and for $R < x\cos\phi$ the torque is negative. As for x near to the circle, where the acting force is the largest, we have $R < x\cos\phi$, I shall write the torque as a negative number -D, where D, because of the appearing singularity, is infinitely large

$$M = \int_0^{2\pi} d\phi \int_R^{\infty} \frac{R^2(R - x\cos\phi)\, dx d\phi}{(R^2 - 2xR\cos\phi + x^2)^{3/2}} = - D. \qquad (27.19)$$

27.5. ACTION OF THE AXIAL CURRENT ON THE CIRCULAR CURRENT.

Before beginning with the calculation, let me note that the torque exerted by the circular current on the axial current obviously is zero as the levers of the forces are null (see fig. 9).

The elementary torque about the z-axis appearing as a result of the action of the axial current element **dr'** on the circular current element dr will be given by formula (27.1). Putting in it (24.3) we obtain

$$\mathbf{dM} = (R^2/r^2)\hat{\rho}\times\cos\psi'\hat{\boldsymbol{\phi}} dr dr'. \qquad (27.20)$$

We have from fig. 9

$$\cos\psi' = - z/r, \qquad r^2 = z^2 + R^2, \qquad (27.21)$$

so that by putting (27.21) into (27.20) we obtain

$$dM = - \frac{R^2 z\, dz\, d\phi}{(z^2 + R^2)^{3/2}}. \qquad (27.22)$$

The elementary torque is obviously negative. For the integral torque we obtain

$$M = - \int_0^{2\pi} d\phi \int_0^{\infty} \frac{R^2\, z\, dz}{(z^2 + R^2)^{3/2}} = - \int_0^{2\pi} R\, d\phi = - 2\pi R. \qquad (27.23)$$

The torque with which the rectangular current acts on the circular current will be the sum of the torques (27.7), (27.18) and (27.23) and is null as it must be.

The torque with which the circular current acts on the rectangular current will be given by the sum of the torques (27.13) and (27.19). As it also must be null, we shall have B = D.

The torque acting on the moving part of Sigalov's experiment (fig. 10) will be the sum of the torques (27.7), (27.13), (27.18) and (27.23). Thus it will be equal to the positive number B. As a matter of fact Sigalov's experiment is a simplified variation of the cemented Barlow disk (see Sect. 47). If the sliding contact will be not at point E but at point F and the circular current will not rotate, Sigalov's experiment will be a simplified variation of the uncemented Barlow disk. As the net torque on the current in the circular wire is null, its rest or rotation is immaterial.

28. THE ROTATING AMPERE BRIDGE (RAB)

The drawing of the circuit which I have called ROTATING AMPERE BRIDGE (RAB) is presented in fig. 14. Current I comes from "infinity" along the upper axial wire PO, flows along the upper rotating and propulsive arms OA and AB with lengths R, along the shoulder BB', then along the lower propulsive and rotating arms B'A' and A'O' and along the lower axial wire O'P' goes to "infinity".

Easily can be seen, taking into account Whittaker's formula (24.3), that the net torques about the z-axis produced by the interaction of the currents in the following wires are null: (i) axial wires and rotating arms, (ii) axial wires and shoulder, (iii) shoulder and propulsive arms, (v) action of propulsive arms on axial wires, (vi) action of shoulder on rotating arms.

Different from zero are only the torques due: (i) to the action of the currents in the axial wires on the currents in the propulsive arms, (ii) to the interaction of the currents in the rotating and propulsive arms, and (iii) to the action of the currents in the rotating arms on the current in the shoulder.

Now I shall calculate the respective torques, omitting also in this section to write the factor I^2/c^2 in the formulas.

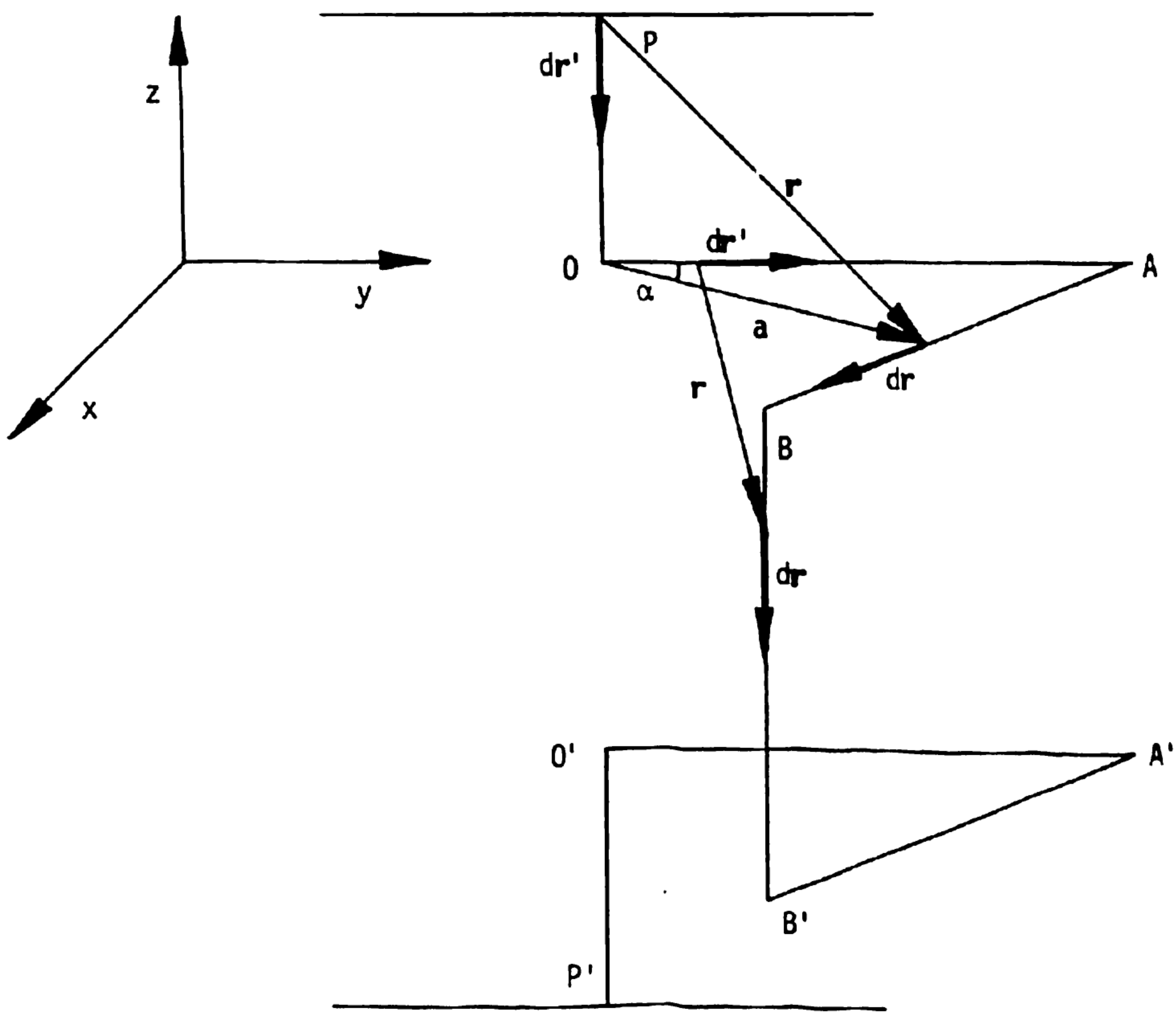

Fig. 14. the rotating Ampere bridge.

28.1. ACTION OF THE AXIAL CURRENT ON THE PROPULSIVE ARM CURRENT.

A current element $I d\mathbf{r}'$ along the axial wire PO acts on a current element $I d\mathbf{r}$ along the propulsive arm AB, to which the vector distance is $\mathbf{r}$, with the elemental force generating torque about the z-axis

$$df = (\mathbf{r}.d\mathbf{r}')d\mathbf{r}/r^3 = \cos(\mathbf{r},d\mathbf{r}')drdr'\hat{\mathbf{x}}/r^2 = z\, dxdz\hat{\mathbf{x}}/(x^2 + z^2 + R^2)^{3/2}. \qquad (28.1)$$

The moment of this force about the z-axis will be

$$d\mathbf{M} = (x\hat{\mathbf{x}} + R\hat{\mathbf{y}})\times z dxdz\hat{\mathbf{x}}/(x^2 + z^2 + R^2)^{3/2} = - Rzdxdy\hat{\mathbf{z}}/(x^2 + z^2 + R^2)^{3/2}. \qquad (28.2)$$

For the z-component of the integral torque we obtain, taking AB = R, PO = ∞,

$$M = - \int_0^R \int_0^\infty Rzdxdz/(x^2 + z^2 + R^2)^{3/2} = - R\int_0^R (x^2 + R^2)^{-1/2}dx = - R\,\mathrm{Arsinh}1. \qquad (28.3)$$

If the shoulder BB' is long enough, we can neglect the torque produced by the action of the axial wire current PO on the current in the propulsive arm B'A'. Thus taking into account also the torque due to the action of the current O'P' on the current B'A', we shall obtain for the z-component of the net torque

$$M_{net} = - 2R\,\mathrm{Arsinh}1 = - 1.7628R. \qquad (28.4)$$

28.2. INTERACTION BETWEEN THE ROTATING ARM CURRENT AND THE PROPULSIVE ARM CURRENT.

Let us calculate first the force with which a current element $d\mathbf{r}'$ of the rotating arm OA acts on a current element $d\mathbf{r}$ of the propulsive arm AB, denoting by $\mathbf{r}$ the vector distance from $d\mathbf{r}'$ to $d\mathbf{r}$ ($\mathbf{r}$ is not shown in fig. 14). According to formula (24.3), in which we exchange the places of the first two terms, we shall have

$$df = \{(\mathbf{r}.d\mathbf{r})d\mathbf{r}' + (\mathbf{r}.d\mathbf{r}')d\mathbf{r}\}/r^3 = \{x\hat{\mathbf{y}} + (R-y)\hat{\mathbf{x}}\}dxdy/r^3. \qquad (28.5)$$

The moment of this force about the z-axis will be, if denoting by $\mathbf{a}$ the vector distance from the axis to the element $d\mathbf{r}$ and by α the angle concluded between the vector $\mathbf{a}$ and the y-axis,

$$dM = \mathbf{a}\times\{x\hat{\mathbf{y}} + (R-y)\hat{\mathbf{x}}\}dxdy/r^3 = a\{x\sin\alpha - (R-y)\cos\alpha\}\hat{\mathbf{z}}dxdy/r^3 = \qquad (28.6)$$

$$\{x^2 - (R-y)R\}\hat{\mathbf{z}}dxdy/\{x^2 + (R-y)^2\}^{3/2}. \qquad (28.6)$$

Let us now calculate the force with which the current element $d\mathbf{r}$ of the propulsive arm AB acts on the current element $d\mathbf{r}'$ of the rotating arm OA, denoting also in this calculation by $\mathbf{r}$ the vector distance <u>from $d\mathbf{r}'$ to $d\mathbf{r}$</u>,

$$df' = \{- (\mathbf{r}.d\mathbf{r}')d\mathbf{r} - (\mathbf{r}.d\mathbf{r})d\mathbf{r}'\}/r^3 = \{- (R-y)\hat{\mathbf{x}} - x\hat{\mathbf{y}}\}dxdy/r^3. \qquad (28.7)$$

The moment of this force about the z-axis will be

$$d\mathbf{M}' = y\hat{\mathbf{y}}\times\{ -(R-y)\hat{\mathbf{x}} - x\hat{\mathbf{y}}\}dxdy/r^3 = y(R-y)\hat{\mathbf{z}}dxdy/\{x^2 + (R-y)^2\}^{3/2}. \qquad (28.8)$$

The net torque due to the <u>interaction</u> of the current elements in the rotating and propulsive arms will be the sum of the torques (28.6) and (28.8)

$$d\mathbf{M}_{net} = dM + dM' = \{x^2 - (R-y)^2\}\hat{\mathbf{z}}dxdy/\{x^2 + (R-y)^2\}^{3/2}. \qquad (28.9)$$

The integral torque produced by the interaction of the currents in the rotating and propulsive arms will be obtained by integrating the elemental torque (28.9) for x in the limits from 0 to R and for y in the limits from 0 to R. If making then the substitution R-y = Y, dx = - dY, we obtain for the z-component of the net torque

$$M_{net} = \int_0^R \int_0^R \{x^2 - (R-y)^2\} dx dy / \{x^2 + (R-y)^2\}^{3/2} = \int_0^R \int_0^R (x^2 - Y^2) dx dY / (x^2 + Y^2)^{3/2} = 0. \qquad (28.10)$$

Thus the net torque due to interaction of the currents in the rotating and propulsive arms is null.

28.3. ACTION OF THE ROTATING ARM CURRENT ON THE SHOULDER CURRENT.

A current element dr' along the rotating arm OA acts on a current element **dr** of the shoulder BB', to which the vector distance is **r**, with the elemental torque

$$\mathbf{dM} = R x \times df, \qquad (28.11)$$

in which we have to put for the elemental force, denoting by z the distance from B to **dr**,

$$df = (\mathbf{r}.\mathbf{dr})\mathbf{dr}'/r^3 = (z/r)y/r^3. \qquad (28.12)$$

Thus we obtain for the z-component of the whole torque, taking BB' = ∞,

$$M = \int_0^R dy \int_0^\infty \frac{Rz\, dz}{\{(R-y)^2 + R^2 + z^2\}^{3/2}} = \int_0^R \frac{dy}{\{R^2 + (R-y)^2\}^{1/2}} = R\,\mathrm{Arsinh} 1. \qquad (28.12)$$

The same torque will be produced by the action of the current in the rotating arm A'O' on the current in the shoulder BB'. Thus for the z-component of the net torque acting on the current in the shoulder we obtain

$$M_{net} = 2R\,\mathrm{Arsinh} 1 = 1.7628R. \qquad (28.13)$$

Comparing formulas (28.4) and (28.13) we see that the net torque due to the interaction of all currents in the rotating Ampere bridge is null.

One can easily see that if the length of the shoulder will be not considered as very long, the net torque acting on RAB will be again zero. In such a case the net torque acting on the shoulder will be less than (28.13) but besides the negative torque (28.4) there will be a positive torque acting on the current AB in the propulsive arm due to the lower axial current O'P'. The relevant calculation gives for the net torque again null result.

As all current elements in RAB are mutually perpendicular, the calculation with Nicolaev's formula will lead to the same result.

Easily can be calculated[22] that also according to Grassmann's formula the torque in RAB must be null.

Ampere's formula which preserves Newton's third law, of course, will lead to a null torque in RAB.

29. ELECTROMOTORS DRIVEN BY VECTOR AND SCALAR MAGNETIC INTENSITIES

The vector and scalar magnetic intensities are defined, respectively, by the second and third formulas (8.6).

If not Whittaker's formula (24.3) but Nicolaev's formula (24.12) will be the right one, the scalar magnetic intensity is to be written not in the simple Whittaker's form (8.6) but in the complicated Nicolaev's form (24.14). Without precising the exact mathematical expression of the scalar magnetic intensity S through the magnetic potential **A** (for the time being when not enough experimental evidence is accumulated), I shall call scalar magnetic intensity this potential force which acts along the test current element and vector magnetic intensity this one which acts at right angles to the test current element. When it will be necessary, I shall present the scalar magnetic intensity preferably in its Whittaker's form.

The ELECTROMAGNETIC MOTORS which are driven by the vector magnetic intensity **B** (such are all electromotors built by humanity in two centuries of electromagnetism) will be called B-MOTORS and the electromagnetic motors which are driven by the scalar magnetic intensity S (see Sects. 58 -60) will be called S-MOTORS.

Here I shall present the most simple S-motor which still I have not constructed, but I have no doubts that it would not work in the predicted way.

We have found in Sect. 27.5 that the torque with which an axial current acts on a circular current (see fig. 9) is given by formula (27.23). As in all formulas of Sect. 27, for brevity's sake, the common factor II'/c^2 was omitted, let us write again this formula in its complete form: Thus the z-component of the torque with which a vertical positive current I' acts on a current I flowing along a circle with radius R in the positive (anti-clockwise) direction is

$$M = - 2\pi II'R/c^2. \qquad (29.1)$$

Let us then construct our S-motor in the following way (fig. 15):

A condenser C with a big capacitance is charged to a high potential. The vertical wire ac, which at its lower end is connected with a big metal sphere, can make successively contact with the positive and negative electrodes of the condenser C. If this contact will be made with a frequency equal to the own frequency of oscillations of the suspended on strings permanent ring magnet, this magnet can be set in oscillations. Indeed, the permanent ring magnet can be presented as two circular currents, I, with radii equal to the internal and external radii of the ring magnet, R_{int} and R_{ext}. The torque acting on these circular currents, for the moment shown in the figure when electrons fly from the left plate of the condenser downwards to the big metal sphere (i.e., when the current is pointing upwards) at the indicated directions of the currents in the magnet (on the internal periphery the current is flowing clockwise and on the external periphery anti-clockwise) will be

$$M_{net} = M_{int} + M_{ext} = (2\pi II'/c^2)(R_{int} - R_{ext}). \qquad (29.2)$$

Thus the motion of the magnet at this laps of time will be negative (clock-wise). At the next laps of time, when the metal sphere will be connected to the right, positive electrode of the big condenser, the motion of the magnet will be positive.

Let now exchange the ring magnet in fig. 15 by a circular wire and let insert in it a source of alternating electric tension with frequency ν. If the frequency with which the wire ac is connected successively to the negative and positive electrodes of the condenser C will be also ν, the circular current wire will begin to rotate. As the moment of force with which the circular current wire acts on the vertical current wire is zero, this experiment will present a patent violation of the angular momentum conservation law.

It is interesting to note that the scalar magnetic intensity with which the electromagnetic system consisting of the driving big condenser C, the wire ca and the big "storage" sphere acts on the circular current can be calculated either as a magnetic effect by the help of the last equation (8.6) or as an electric effect by the help of equation (8.10). The force on the circular current will act in the direction of the current when $\mathrm{div}\mathbf{A} < 0$, i.e., $\partial\Phi/\partial t > 0$, or against the direction of the current when $\mathrm{div}\mathbf{A} > 0$, i.e., $\partial\Phi/\partial t < 0$.

These childichaly simple and clear effects are absolutely unknown to offial physics.

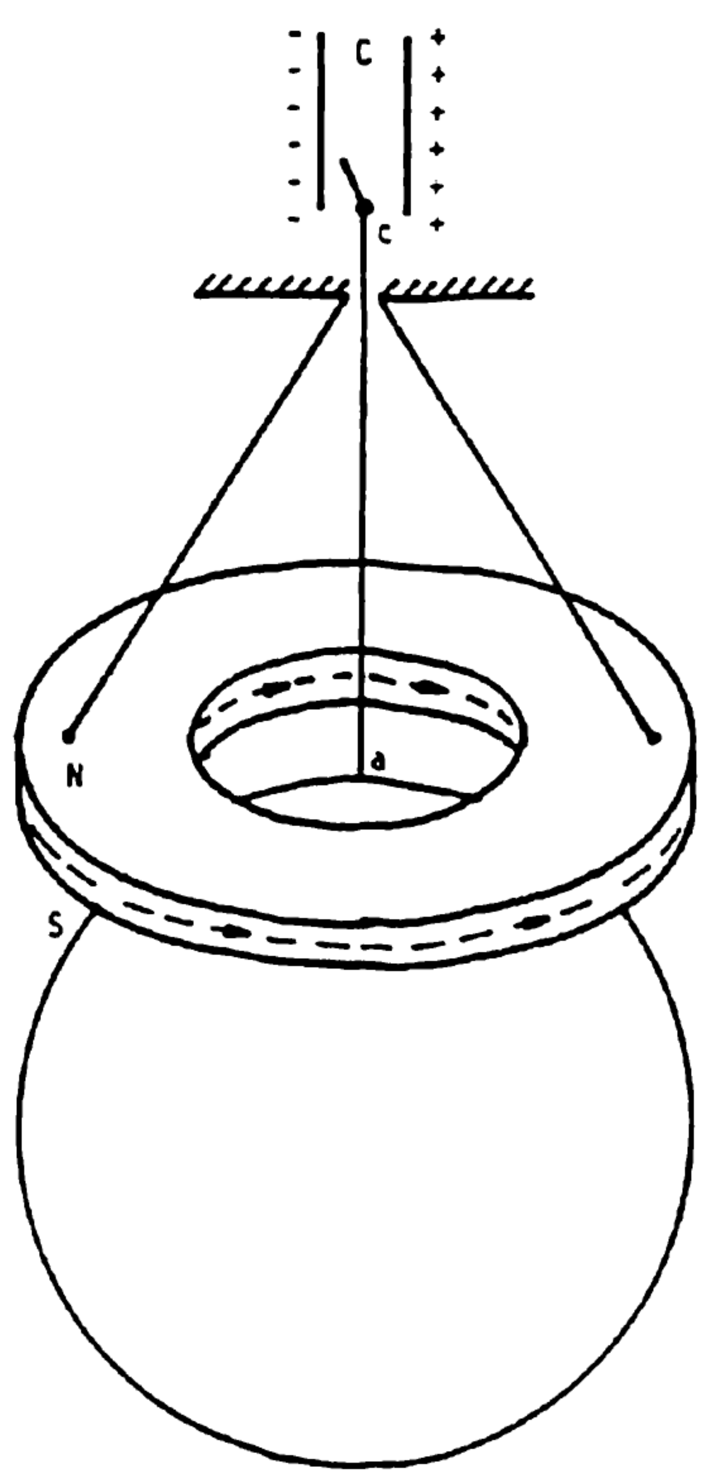

Fig. 6

Fig. 15. S-motor with interrupted current.

Let us make these two types of calculation for the experiment shown in fig. 15, making use also of fig. 9. We suppose that the current wire ac is infinitely long and that a constant current I' flows along it from point a (where there is a big "storage" sphere charged with positive charges) to point c (where there is another "storage" sphere charged with negative charges). The magnetic potential generated by the current I' along the circular loop with radius R will be

$$A = \int_0^\infty I' dz\hat{z}/cr = (I'/c)\int_0^\infty dz\hat{z}/(R^2 + z^2)^{1/2}. \qquad (29.3)$$

The scalar magnetic intensity generated by this vertical current along the circular loop will be

$$S = - \mathrm{div}A = - \partial A/\partial r = - (I'/c)\int_0^\infty z dz/(R^2 + z^2)^{3/2} = - I'/cR. \qquad (29.4)$$

We shall obtain the same value for the scalar magnetic intensity, if calculating it according to formula (8.10). To make the calculation more simple, let calculate S in the equatorial plane of the storage sphere at a distance R from its center. The potential of the charges q on the sphere at a distance R from the center is $\Phi = q/R$, independently of the radius of the sphere[5]. When the current extracting charges from the storage sphere is I', for a time Δt the extracted charges will be $I'\Delta t$ and we shall have for the scalar magnetic intensity

$$S = \partial\Phi/c\partial t = (1/c)\Delta\Phi/\Delta t = \frac{(q - I'\Delta t)/R - q/R}{c\Delta t} = - I'/cR, \qquad (29.5)$$

what is exactly the value (29.4).

Let me note that yet Grassmann[18] pointed out that the observation of the action of open currents on other currents (current elements) is of a high importance. For my big surprise, to the best of my knowledge, no such quantitative observations have been done in the 150 years after Grassmann. Here I should like to cite some remarkable lines of Grassmann:[18] (p. 14)

> Oberhaupt ist klar, daß eine Entscheidung zwischen beiden Theorien (Ampere's and Grassmann's theories), da die Wirkung, welche geschloßene Ströme üben, nach beiden dieselbe ist, nur möglich ist, wenn man die Wirkung betrachtet, welche ein begränzter Strom übt... Der begränzte Strom würde daher so hervorzurufen sein, daß man zuerst etwa zwei Kugeln mit entgegengesetzter Elektrizität möglichst stark lüde, und sie dann nach der Ladung (nicht während derselben) in leitende Verbindung brächte. Dann hätte man die Wirkung dieses begränzten Stromes auf irgend einen elektrischen Strom oder besser auf einen Magneten zu beobachten, und die Anordnung dabei so zu treffen, daß die Wirkungen nach beiden Theorien möglichst verschieden erfolgen.

If someone had followed Grassmann's advice and had done the experiment shown in fig. 15, one would had observed the rotation of circular current many and many years ago, and the wrong dogma that the magnetic force acting on a current element must be always at right angles to the element would not survive all these years. Neither Maxwell's dogma about the closed currents could then survive.

Now I shall reveal a very interesting aspect of the S-motors, namely that not back but forth tension is induced at their rotation.

If the current along the circular loop is flowing anti-clockwise (as in fig. 9), the forces acting on the current conducting charges, according to the fourth formula (21.1) - as well as according to formula (24.15) - will be directed against their velocities, so that the circular wire will begin to rotate in a clockwise direction. At this motion, all positive charges in the wire which can become current conducting charges will obtain a low convection velocity in a clock-wise direction. The scalar magnetic intensity (29.4) will begin to act on these convected charges, according to the fourth formula (21.1), with an electromotive force opposite to their velocity, i.e., with a force pointing along the direction of the initial driving current.

The force acting on a unit convected positive charge will be the induced electric intensity (see again formula (29.4))

$$\mathbf{E}_{ind} = (\mathbf{v}/c)S = \Omega RS\mathbf{n}/c^2 = -\ \Omega I'\mathbf{n}/c^2, \qquad (29.6)$$

where Ω is the angular velocity of rotation of the circular wire and $\mathbf{n}$ is the unit vector at any single point of the wire pointing along its linear rotational velocity, i.e., against the direction of the initial driving current. Thus the electric intensity induced by the scalar magnetic intensity is directed along the driving current and I call it INDUCED FORTH ELECTRIC INTENSITY.

The induced electric tension will be

$$U_{ind} = \int_{2\pi R} \mathbf{E}_{ind}\cdot d\mathbf{r} = -\ (2\pi/c^2)\ \Omega RI'. \qquad (29.7)$$

and will also act in anti-clockwise direction, i.e., will have the same direction as the driving electric tension, U_{dr}, and I call it INDUCED FORTH ELECTRIC TENSION.

We know that the tension induced in motors driven by a vector magnetic intensity, $\mathbf{B}$, is always opposite to the driving tension and for this reason it is called INDUCED BACK ELECTRIC TENSION. And one can immediately show why in B-motors a back electric tension is induced:

Let us have a current element $Id\mathbf{r}$ put in a vector magnetic field $\mathbf{B}$ which is perpendicular to $d\mathbf{r}$. The force acting on this current element, according to the third formula (21.1) is

$$d\mathbf{f}_{wire} = (Id\mathbf{r}/c)\times\mathbf{B}. \qquad (29.8)$$

The velocity $\mathbf{v}$ acquired by the wire will have the direction $d\mathbf{f}_{wire}$ which is $d\mathbf{r}\times\mathbf{B}/drB$, and the induced electric intensity acting on the convected charges will be, again according to the third formula (21.1),

$$c\mathbf{E}_{ind} = v(d\mathbf{r}\times\mathbf{B}/drB)\times\mathbf{B} = -\ (v/drB)\mathbf{B}\times(d\mathbf{r}\times\mathbf{B}) = -\ (v/drB)B^2 d\mathbf{r} = -\ vB(d\mathbf{r}/dr), \qquad (29.9)$$

i.e., it will be directed against the driving electric intensity (and tension) which acts in the direction $d\mathbf{r}/dr$.

After having presented the "mechanism" according to which a forth electric tension is induced in S-motors and a back electric tension is induced in B-motors, let us make a more detailed comparison between a B-motor and an S-motor.

Let us assume that both motors have the same ohmic resistance R_0 and that they are driven by equal driving tensions U_{dr}. Thus the rest current in both motors will be the same $I_{rest} = U_{dr}/R_0$.

If we let the B-motor rotate, it will acquire such an angular velocity Ω that its friction power $P_{fr} = \Omega M_{fr}$, where M_{fr} is the friction torque at the angular velocity Ω, will become equal to the induced back power $P_{ind} = IU_{ind}$, where U_{ind} is the indced back tension and I is the current in the motor at the angular velocity Ω.

Indeed, let us assume, for simplicity, that the motor is a Barlow disk (see Sect. 47) with radius R in which the cylindrical magnetic field with intensity B is generated by a cylindrical magnet. The driving torque is produced by the interaction of B and the current I which flows along the disk's radius. If we consider only one current element Idr at a distance r from the center, the driving torque produced by its interaction with B will be $dM_{dr} = rdf = rIdrB/c$, where $df = IdrB/c$ is the force acting on the current element. The motor will stop to increase its angular velocity when the sum of all these elementary torques will become equal to the friction torque M_{fr}. At the "equilibrium" angular velocity Ω, when the current in the circuit will be I, we shall have

$$\Omega M_{fr} = \Omega \int_0^R dM_{fr} = \Omega \int_0^R rIdrB/c = I \int_0^R vBdr/c = IU_{ind}, \qquad (29.10)$$

where v is the velocity of the disk's parts with radius r and U_{ind} is the induced back electric tension. For the current we shall have $I = (U_{dr} - U_{ind})/R_0$. At rest of the disk the power $P_{rest} = I_{rest}U_{dr} = I^2_{rest}R_0$ will be released as heat. At rotation of the disk the power $P = I(U_{dr} - U_{ind}) = I^2R_0$ will be released as heat and the power $P_{mech} = IU_{ind}$ will be delivered as mechanical power overwhelming the friction. The power delivered by the driving electric source $P_{dr} = IU_{dr}$ will be the sum of the last two powers.

If we let the S-motor rotate, it will acquire such an angular velocity Ω that its friction power $P_{fr} = M_{fr}$ will become equal to the iduced forth power $P_{ind} = IU_{ind}$.

Indeed, let us assume, for simplicity, that our motor is of the kind of the motor shown in fig. 9, assuming that at the point a there is a huge store of positive charges and at point c there is a huge store of negative charges, so that certain time a constant current I' flows from point a to point c. The driving torque produced by the action of the scalar magnetic intensity S on the current along the circular loop will be

$$M_{dr} = \int_{2\pi R} R\hat{\rho}\times df_{whit}, \qquad (29.11)$$

where (see (29.4))

$$df_{whit} = IdrS/c = - II'dr/c^2R \qquad (29.12)$$

is the force acting on the current element Idr. Putting (29.12) into (29.11), we obtain for the z-component of the driving torque

$$M_{dr} = -\int_{2\pi R} II'dr/c^2 = -2\pi RII'/c^2. \qquad (29.13)$$

The motor will stop to increase its angular velocity when its driving torque will become equal to its friction torque. At such an "equilibrium" angular velocity Ω we shall have (see (29.7)), noting that M_{fr} and M_{dr}, at the "equilibrium" angular velocity Ω, are equal but oppositely directed,

$$\Omega M_{fr} = \Omega M_{dr} = -(2\pi/c^2)\Omega RII' = IU_{ind}. \qquad (29.14)$$

At such a stationary rotation the power $P = I(U_{dr} + U_{ind}) = I^2R_0$ will be released as heat and the power $P_{mech} = IU_{ind}$ will be delivered as mechanical power overwhelming the friction. The power delivered by the driving electric source $P_{dr} = IU_{dr}$ will be the difference of these two powers.

The driving torque of the B-motor is the largest at rest of the motor and reaches its minimum at the angular velocity Ω. The driving torque of the S-motor is the less at rest and reaches its maximum at the angular velocity Ω.

If the friction power ΩM_{fr} will always remain less than the mechanical power IU_{ind}, the S-motor will steadily increase its angular velocity until the destruction of the motor by the appearing centrifugal forces. Thus the S-motor violates the energy conservation law.

A B-motor can be run as a GENERATOR (machine generating electric tension and eventually electric current and power) if applying to it a mechanical torque. The mechanical torque which appears in a B-GENERATOR, because of the interaction of the induced current with the B-field, is always directed oppositely to the driving mechanical torque and brakes the rotation. In every conventional B-generator the produced electrical power is equal to the mechanical power lost by the source of mechanical energy. Let me note, however, that I have constructed B-generators where quite the whole produced power is "free", i.e., produced from nothing; such are my non-braking B-generator MAMIN COLIU (Sect. 53) and the self-accelerating generator VENETIN COLIU (Sect. 54).

The considered above S-motor can also be run as a generator, applying to it a mechanical torque. The mechanical torque which appears in an S-GENERATOR, because of the interaction of the induced current with the S-field, is always directed in the direction of the driving mechanical torque and supports the rotation. The produced electric power in the S-generator is equal to the mechanical power gained by the source of mechanical energy.

If Whittaker's formula is the right one, a scalar magnetic field can be not produced by closed current loops, as the divergence of the magnetic potential produced by a closed current loop is zero according to Whittaker's formula. As, however, it is very likely that Nicolaev's formula is the right one, S-motors and S-generators can be "driven" by closed currents. Such machines are considered in Sects. 58 -60.

30. QUASI-STATIONARY ELECTROMAGNETIC SYSTEMS

I make the following classification of the material systems (see also Sect. 9):

1. A material sytsem is called STATIC if there is such a frame of reference with respect to which its particles remain at rest. The image (see Sect. 2) of a static system remains the same in time.

2. A material system is called QUASI-STATIC if its images remain the same in time but there is no such a frame of reference with respect to which its particles remain motionless. According to this definition, the particles of a quasi-static system can move with respect to each other, but in the direction of their velocities they must be placed closely enough and they must have the same character, so that they may be distinguished by their serial numbers only. If we do not pay attention to their serial numbers, such a system will, in different moments of time, create the same image in our mind. The moving points of a quasi-static system always form ring-shaped current tubes.

3. A material system is called STATIONARY if some of its characteristics remain constant in time. The quasi-static system represents the most simple stationary system because the whole complex of characteristics, namely its image, remains constant in time.

4. A material system is called QUASI-STATIONARY if some of its characteristics change insignificantly in time or in certain specific time interval.

5. A material system is called DYNAMIC if its images change in time.

6. A material system is called PERIODIC if its images repeat themselves regularly after some time interval. This time interval is called PERIOD.

7. A material system is called QUASI-PERIODIC if its images repeat themselves after some time interval but not completely; however, after sufficiently long period of time (i.e., with the increase of the number of the "quasi-periods") the image of the system approaches closely enough its initial image.

The field of static and quasi-static systems of electric charges is called a CONSTANT ELECTROMAGNETIC FIELD.

Let us consider a system of electric charges which generates the potentials Φ and $\mathbf{A}$ (given by formulas (8.1)) in the different space points.

1. If

$$\partial\Phi/\partial t = 0, \qquad \mathbf{A} = 0, \tag{30.1}$$

the system is static.

2. If

$$\partial\Phi/\partial t = 0, \qquad \partial\mathbf{A}/\partial t = 0, \tag{30.2}$$

the system is quasi static or stationary.

3. If

$$\partial\Phi/\partial t \neq 0, \qquad \partial\mathbf{A}/\partial t \neq 0, \tag{30.3}$$

but we can assume

$$\partial^2\Phi/\partial t^2 = 0, \qquad \partial^2\mathbf{A}/\partial t^2 = 0, \tag{30.4}$$

the system is quasi-stationary.

The conditions (30.4) can be fulfilled strictly only if Φ and **A** are linear functions of time (for example a circular current which constantly increases its radius). If the system is periodic, the conditions (30.4) cannot be fulfilled. But if the periodic change is slow and for long enough time intervals we can accept that Φ and A are linear functions of time, we can accept the system to be quasi-stationary.

Usually if the shortest period of the system T_{min} is much larger than the time $t = D_{max}/c$, where D_{max} is the largest size of the system, the system is quasi-stationary.

Another criterion for accepting an electromagnetic system to be quasi-stationary is the following: The effects due to the accelerations (second time derivatives) of the charges (i.e., the radiation of the charges) must be feeble and thus can be neglected.

For a quasi-stationary system not equations (9.16) but equations (9.15) are valid. Let us write them again

$$\Delta\Phi \equiv \mathrm{div}(\mathrm{grad}\Phi) = -4\pi Q, \qquad \Delta\mathbf{A} \equiv \mathrm{grad}(\mathrm{div}\mathbf{A}) - \mathrm{rot}(\mathrm{rot}\mathbf{A}) = -4\pi\mathbf{J}. \qquad (30.5)$$

As I showed in Sect. 9, these equations are trivial mathematical results of the definition equalities (8.1) for the electric and magnetic potentials and equalities (9.14) for the charge and current densities.

Another trivial result of equations (8.1) is the equation of potential connection (8.8) which I write here again

$$\mathrm{div}\mathbf{A} = -\partial\Phi/c\partial t. \qquad (30.6)$$

Let us write again the first notation (21.1) and the second notation (8.6)

$$\mathbf{E}_{coul} = -\mathrm{grad}\Phi, \qquad \mathbf{B} = \mathrm{rot}\mathbf{A}, \qquad (30.7)$$

called Coulomb electric intensity and magnetic intensity.

If we rewrite the second equation (21.1) and we take divergence from the second expression (30.7), we obtain

$$\mathbf{E}_{tr} = -\partial\mathbf{A}/c\partial t, \qquad \mathrm{div}(\mathrm{rot}\mathbf{A}) = 0, \qquad (30.8)$$

or

$$\mathrm{rot}\mathbf{E}_{tr} = -\partial\mathbf{B}/c\partial t, \qquad \mathrm{div}\mathbf{B} = 0. \qquad (30.9)$$

If we substitute (30.6) and the second expression (30.7) into the second equation (30.5) and if we rewrite the first expression (30.5), we shall have

$$\mathrm{rot}\mathbf{B} = -\partial(\mathrm{grad}\Phi)/c\partial t + 4\pi\mathbf{J}, \qquad \mathrm{div}(\mathrm{grad}\Phi) = -4\pi Q, \quad (30.10)$$

or

$$\mathrm{rot}\mathbf{B} = \partial\mathbf{E}_{coul}/c\partial t + 4\pi\mathbf{J}, \qquad \mathrm{div}\mathbf{E}_{coul} = 4\pi Q. \qquad (30.11)$$

Equations (30.8) and (30.10) are the Maxwell-Lorentz equations for a quasi-stationary system of electric charges in their most logical form.

Equations (30.9) and (30.11) are the Maxwell-Lorentz equations for a quasi-stationary system in their usual form. It is extremely important to note that $\mathbf{E}_{tr}$ in the first equation (30.9) is completely different from E_{coul} in the first equation (30.11). These two electric intensities have noting in common. However B in the first equation (30.9) and B in the first equation (30.11) is one and the same quantity.

Official physics defends the opinion that a magnetic field can generate elecric field and electric field can generate magnetic field. This is a complete nonsense (this view-point is defended also by Jefimenko in his new book "Causality, electromagnetic induction and gravitation"(Electret Scientific Company, Star City, WV 26505, USA, 1992)). The electric and magnetic intensities are determined (and defined!) by the potentials and only by the potentials.

Now I shall examine the highly controversial problem about the "DISPLACEMENT CURRENT" (see Sect. 13). I shall show that there is nothing puzzling here if this notion will be rightly understood.

Maxwell supposed that if a conduction current becomes interrupted at the plates of a condenser, between those plates a current with density (13.12) "flows", called "displacement current". Maxwell supposed that displacement current has the same magnetic character as conduction current with the same density, i.e., that it acts with potential magnetic forces on other currents and reacts with kinetic forces against the potential magnetic action of other currents. And Maxwell supposed (or such was rather the interpretation of his epigones) that all this is done by the hypothetical current "flowing between the plates of the condenser". This is absolutely not true.

It is obvious that such a displacement current cannot react with kinetic forces against the action of other currents, as it flows in vacuum, and neither the Lord is able to set vacuum in motion. On other side vacuum cannot act with potential forces on other currents as vacuum is vacuum ("a rose is a rose, is a rose, is a rose").

To understand the essence of the displacement current, let us consider not the differential equation (30.11) but the integral equation (13.11), rewriting it for a quasi-stationary system

$$\oint_L \mathbf{B}.d\mathbf{r} = (\partial/c\partial t)\int_S \mathbf{E}_{coul}.d\mathbf{S} + (4\pi/c)\int_S \mathbf{J}.d\mathbf{S}. \qquad (30.12)$$

The magnetic intensity is generated by the currents in whole space. Meanwhile in (30.12) the linear integral of B along the closed loop L is related only to the conduction currents crossing the surface S. If from both sides of S there are condenser's plates which interrupt conduction currents, these interrupted currents generate such an electric intensity field E_{coul} between the condenser's plates that

$$\oint_L \mathbf{B}.d\mathbf{r} = (\partial/c\partial t)\int_S \mathbf{E}_{coul}.d\mathbf{S}. \qquad (30.13)$$

Thus it is not the changing electric field $\partial E_{coul}/\partial t$ which generates B. The integral on the right side of (30.13) gives simply information about the quantity of conduction current interrupted on the surface S. Consequently the magnetic intensity calculated by formula (30.13) is generated by charges flowing to the condenser's plates and these charges react with kinetic forces to the action of other currents flowing between the condenser's plates or outside.

If $\partial E_{coul}/\partial t = 0$, formula (30.12) shows that $\oint B.dr$ is determined only by the quantity of current crossing the surface. This is true. But when one begins to calculate to find **B**, one sees that one has to take into account the currents in whole space. The displacement current term in (30.12) indicates that when making integral calculations to find **B** one has to take into account also the interrupted by the surface S currents.

That's all about the displacement current!

Let us now assume that the considered electromagnetic system consists not only of charges (free or in conductors) but also of dielectrics and magnetics. In such a case the Maxwell-Lorentz equations (30.9) and (30.11) are to be written in the form

$$\mathrm{rot}E_{tr} = -\,\partial B/c\partial t, \qquad \mathrm{div}B = 0, \qquad (30.14)$$

$$\mathrm{rot}H = \partial D/c\partial t + 4\pi J, \qquad \mathrm{div}D = 4\pi Q. \qquad (30.15)$$

Now, if there is a condenser between whose plates a dielectric with permittivity ε is put, between these plates a POLARIZATION CURRENT will flow with density

$$J_{pol} = \partial(D-E)/c\partial t = (\varepsilon - 1)\partial E/c\partial t. \qquad (30.16)$$

This current does not transfer charges from one plate of the condenser to the other, as the case will be if the plates will be connected by a wire. Because of the orientation (or polarization) of the molecular electric dipoles along the field of the acting electric intensity **E**, generated by the charges on the plates, it seems that charges have been transferred, but, as a matter of fact, charges have not been transferred.

The same phenomenon appears also when there is vacuum between the plates: as the charges coming to one of the plates repel by electrostatic induction charges of the same sign from the other plate, it also seems that charges have been transferred. Thus there are many common features between polarization current and displacement current, and some people call also the polarization current "displacement current". I, however, rigirously separate them. In any case, both the displacement and polarization currents do not act with potential magnetic forces on other currents and do not react with kinetic forces against the potential action of other currents. I confirmed these assertions experimentally (see Sects. 61 and 62).

31. ELECTRIC DIPOLE MOMENT

Let us consider the constant electric field of a stationary system of charges at large distances from the system, that is, at distances large compared with the dimensions of the system.

We introduce a frame of reference with its origin somewhere in the system of charges. Let us denote the radius vector of the reference point by $\mathbf{r}$ and the radius vector of the various charges by $\mathbf{r}_i$. According to the first formula (8.1), the electric potential generated by the system at the reference point will be

$$\Phi = \sum_{i=1}^{n} q_i/R_i = \sum_{i=1}^{n} q_i/|\mathbf{r} - \mathbf{r}_i|, \qquad (31.1)$$

where

$$\mathbf{R}_i = \mathbf{r} - \mathbf{r}_i \qquad (31.2)$$

is the vector from the charge q_i to the reference point.

Let us investigate expression (31.1) for large r, i.e., for $r \gg r_i$. To do this, let us expand (31.1) as power series in $\mathbf{r}_i$, retaining only the terms linear in $\mathbf{r}_i$,

$$\Phi(|\mathbf{r} - \mathbf{r}_i|) = \Phi(r) - \sum_{i=1}^{n} \{\partial\Phi(r)/\partial\mathbf{r}\}.\mathbf{r}_i = \sum_{i=1}^{n} q_i/r_i - \mathrm{grad}(1/r).\sum_{i=1}^{n} q_i\mathbf{r}_i. \qquad (31.3)$$

If we denote the total charge by

$$q = \sum_{i=1}^{n} q_i, \qquad (31.4)$$

formula (31.3) can be written

$$\Phi = q/r + \mathbf{d}.\mathbf{r}/r^3, \qquad (31.5)$$

where the sum

$$\mathbf{d} = \sum_{i=1}^{n} q_i\mathbf{r}_i \qquad (31.6)$$

is called ELECTRIC DIPOLE MOMENT of the system of charges.

It is important to note that if the sum of all charges is equal to zero

$$q = \sum_{i=1}^{n} q_i = 0, \qquad (31.7)$$

then the dipole moment does not depend on the choice of the frame's origin. Indeed, the radius vectors $\mathbf{r}_i$ and $\mathbf{r}_i'$ of one and the same charge in two different frames of reference, K and K', are related by the formula

$$\mathbf{r}_i = \mathbf{R} + \mathbf{r}_i', \qquad (31.8)$$

where $\mathbf{R}$ is a constant vector, representing the radius vector of the origin of K' in K. Substituting (31.8) into (31.6) and taking into account (31.7), we obtain $\mathbf{d} = \mathbf{d}'$.

Under the condition (31.7), the electric potential in formula (31.5) becomes

$$\Phi = \mathbf{d}.\mathbf{r}/r^3. \qquad (31.9)$$

The electric intensity, according to the first formula (21.1), will be

$$\mathbf{E} = -\,\mathrm{grad}(\mathbf{d}.\mathbf{r}/r^3) = -\,(1/r)\mathrm{grad}(\mathbf{d}.\mathbf{r}) - (\mathbf{d}.\mathbf{r})\mathrm{grad}(1/r^3). \qquad (31.10)$$

Keeping in mind that $\mathbf{d}$ is a constant vector, we shall have (see p. 6)

$$\mathrm{grad}(\mathbf{d}.\mathbf{r}) = \mathbf{d}, \qquad (31.11)$$

so that

$$E = \{3(d.r)r - r^2 d\}/r^5. \tag{31.12}$$

If we shall expand Φ in (31.3) to higher orders in r_i, we shall obtain other multipole moments. The moment which corresponds to the second order terms in the expansion of Φ is called ELECTRIC QUADRUPOLE MOMENT. Two nearly located opposite charges are called ELECTRIC DIPOLE.

32. MAGNETIC DIPOLE MOMENT

Let us consider the constant magnetic field of a stationary system at large distances from the system.

As in the previous section, we introduce a frame of reference with its origin somewhere in the system of charges. Again we denote the radius vector of the reference point by **r** and the radius vectors of the various charges by r_i. According to the second formula (8.1), the magnetic potential generated by the system at the reference point will be

$$\mathbf{A} = \sum_{i=1}^{n} q_i \mathbf{v}_i / cR_i = \sum_{i=1}^{n} q_i \mathbf{v}_i / c|\mathbf{r} - \mathbf{r}_i|. \tag{32.1}$$

Making the assumption $r \gg r_i$ and expanding (32.1) as a power series to within terms of first order in r_i, we obtain

$$A(|\mathbf{r} - \mathbf{r}_i|) = (1/cr)\sum_{i=1}^{n} q_i \mathbf{v}_i - (1/c)\sum_{i=1}^{n} q_i \mathbf{v}_i \{\mathrm{grad}(1/r).\mathbf{r}_i\}. \tag{32.2}$$

As all currents in the system are closed, the first term on the right will be equal to zero and we shall have

$$\mathbf{A} = (1/cr^3)\sum_{i=1}^{n} q_i v_i (\mathbf{r}_i.\mathbf{r}). \tag{32.3}$$

Taking into account that $v_i = dr_i/dt$ and that **r** is a constant vector, we can write

$$\sum_{i=1}^{n} q_i \mathbf{v}_i (\mathbf{r}_i.\mathbf{r}) = \frac{1}{2}\frac{d}{dt}\{\sum_{i=1}^{n} q_i \mathbf{r}_i (\mathbf{r}_i.\mathbf{r})\} + \frac{1}{2}\sum_{i=1}^{n} q_i \{\mathbf{v}_i(\mathbf{r}_i.\mathbf{r}) - \mathbf{r}_i(\mathbf{v}_i.\mathbf{r})\}. \tag{32.4}$$

If we average this equation in time, the first term on the right side will give zero as a total time derivative of a limited quantity. Thus introducing the quantity

$$\mathbf{m} = (1/2c)\sum_{i=1}^{n} q_i (\mathbf{r}_i \times \mathbf{v}_i) = (1/2c)\sum_{i=1}^{n} \mathbf{r}_i \times \mathbf{j}_i, \tag{32.5}$$

which is called MAGNETIC (DIPOLE) MOMENT of the system of charges, we can present the magnetic potential (32.3) in the form

$$\mathbf{A} = \mathbf{m} \times \mathbf{r}/r^3. \tag{32.6}$$

The magnetic intensity, according to the second formula (8.6), will be (see p. 6

$$\mathbf{B} = \mathrm{rot}(\mathbf{m} \times \mathbf{r}/r^3) = \mathbf{m}\,\mathrm{div}(\mathbf{r}/r^3) - (\mathbf{m}.\mathrm{grad})(\mathbf{r}/r^3). \tag{32.7}$$

First we have (see again p. 6)

$$\mathrm{div}\frac{\mathbf{r}}{r^3} = \frac{1}{r^3}\,\mathrm{div}\mathbf{r} + \mathbf{r}.\mathrm{grad}\frac{1}{r^3} = \frac{3}{r^3} - 3\,\frac{\mathbf{r}.\mathbf{r}}{r^5} = 0, \qquad (32.8)$$

and then

$$(\mathbf{m}.\mathrm{grad})\frac{\mathbf{r}}{r^3} = \frac{1}{r^3}(\mathbf{m}.\mathrm{grad})\mathbf{r} + \mathbf{r}(\mathbf{m}.\mathrm{grad}\frac{1}{r^3}) = \frac{\mathbf{m}}{r^3} - \frac{3\mathbf{r}(\mathbf{m}.\mathbf{r})}{r^5}. \qquad (32.9)$$

Thus for the magnetic intensity (32.7) we obtain

$$\mathbf{B} = \{3(\mathbf{m}.\mathbf{r})\mathbf{r} - r^2\mathbf{m}\}/r^5. \qquad (32.10)$$

We see that the magnetic intensity is expressed in terms of the magnetic moment by the same formula by which the electric intensity is expressed in terms of the electric dipole moment (cf. formula (21.12)).

The magnetic moment of the electron is called MAGNETON OF BOHR and has the value

$$\mathbf{m}_e = q_e h/4\pi m_e c, \qquad (32.11)$$

where q_e and m_e are the charge and the mass of the electron, h is the Planck constant (see Sect. 2) and c is the velocity of light.

The formula for the magneton of Bohr can easily be obtained from formula (32.5) which I shall write in the form

$$\mathbf{m}_e = (1/2c)\mathbf{r}\times q_e\mathbf{v}, \qquad (32.12)$$

considering the charge of the electron (and its mass, too) as a ring with radius r rotating with a velocity **v**. Multiplying and dividing the right side of (32.12) by m_e and taking into account that the angular momentum (the spin) of the electron is

$$|\mathbf{r}\times m_e\mathbf{v}| = h/2\pi, \qquad (32.13)$$

we obtain readily formula (32.11).

IV. HIGH-ACCELERATION ELECTROMAGNETISM

33. INTRODUCTION

In Chapter III the accelerations of the charges were assumed to be small and have been neglected. In this chapter I shall not assume the accelerations of the charges as negligibly small. Thus in this chapter the most general dynamic system of electric charges will be considered.

As it will be shown, charges moving with acceleration radiate energy. The radiated energy is emitted in the form of energetic quanta which are called PHOTONS (with more precision - see beneath - ELECTROMAGNETIC PHOTONS).

The photons always propagate with the velocity c (in absolute space!). The universal masses of the photons are equal to zero, so that their universal space and time momenta are always equal to zero and only their proper space and time momenta are different from zero.

The proper space and time momenta of the photons are very small quantities and one can observe with macroscopic instruments only the collective action of many photons. When observing the flux of many photons, as the latter may interfere (see axiom III), the observer remains with the impression that high-accelerated electromagnetic systems radiate waves, which are called ELECTROMAGNETIC WAVES. However with microscopic instruments, i.e., with particles, one can observe the action of single photons. Thus the assertion "photons are at the same time particles and waves" is wrong. The photons are particles, but these particles can interfere if at the moment of observation the distance between them is less than their proper wavelength (see axiom III).

When masses move with acceleration radiation of GRAVIMAGRETIC PHOTONS is to be expected. I shall show, however, that the radiated gravitational and magretic intensities are so feeble that the detection of gravimagretic photons (waves) is highly improbable.

In high-acceleration electromagnetism I shall ignore the scalar magnetic intensity. Until the present time experiments demonstrating the existence of high-acceleration effects due to the scalar magnetic intensity (SCALAR ELECTROMAGNETIC WAVES) have not been reported. Nicolaev tries to persuade me that he has observed (see "Deutsche Physik", 2(8), 24, 1993)) the existence of scalar electromagnetic waves but, as I show in my comments to his article, his experiments are not convincing me.

34. THE ELECTRIC AND MAGNETIC INTENSITY FIELDS OF AN ACCELERATED CHARGE

To obtain the electric and magnetic intensities generated by a particle moving with acceleration, we have to put in the definition equalities for the electric and magnetic intensities

$$E = - \text{grad}\Phi - \partial A / c\partial t, \qquad B = \text{rot}A \qquad (34.1)$$

the electric and magnetic potentials of the particle

$$\Phi = q/r, \qquad A = qv/cr. \qquad (34.2)$$

However, as information cannot be transferred momentarily, the observation electric and magnetic potentials are to be expressed through the advanced and retarded elements of motion (see Sect. 11).

In fig. 1 the reference point P, for which we wish to know the electric and magnetic intensities at the moment of observation t, is taken at the frame's origin. The charge q generating the potentials and consequently the intensities is shown moving with a constant velocity **v**, but we shall assume now that this velocity is not constant, i.e., that the charge moves with acceleration.

Let us assume that at the observation moment t the charge is at point Q, called observation position. Information about the charge's velocity and acceleration can be obtained at P at the observation moment $t = t' + r'/c = t" - r"/c$, if at the advanced moment t' a signal moving with the velocity c will be sent with this information from the advanced position Q' towards P, or if at the retarded moment t" a signal moving with the velocity c will be sent with this information back in time from the retarded position Q" towards P (so that this signal will reach P at the moment t which is before the moment t"). My second axiom asserts that time has no the quality "reversibility", but "mathematics" does not know this!

The distances r', r and r" are, respectively, the advanced, observation and retarded distances, and the angles θ', θ, θ" between the charge's velocity **v** and the line joining the charge with the reference point (whose unit vectors are **n'**, **n**, **n"**) are, respectively, the advanced, observation and retarded angles.

I repeat (see Sect. 10.2) that official physics, proceeding from the wrong concept that the electromagnetic interactions "propagate" with the velocity c, calls all topsyturvy, i.e., official physics calls the advanced elements "retarded" and the retarded elements (to which it does not pay much attention) "advanced". I shall use only my terminology.

First I shall make the calculation when the observation elements are presented by the advanced elements and then by the retarded ones. As the character of light propagation is not Newton-aether but Marinov-aether, the potentials must be taken in their Lienard-Wiechert forms (see formulas (11.3)).

34.1. CALCULATION WITH THE ADVANCED ELEMENTS OF MOTION.

The observation Lienard-Wiechert potentials expressed through the advanced elements are

$$\Phi = \frac{q}{r'(1 - \mathbf{n}'.\mathbf{v}/c)}, \qquad \mathbf{A} = \frac{q\mathbf{v}}{cr'(1 - \mathbf{n}'.\mathbf{v}/c)}. \qquad (34.3)$$

The velocity in the denominators is a certain middle velocity between the advanced velocity **v**' and the observation velocity **v**, so that moving with this velocity in the time

$t - t' = r'/c$, the charge covers the distance Q'Q. As this velocity appears only in corrective terms in the final result, we can take for it the advanced as well as the observation velocity. The velocity in the nominator of **A** is the observation velocity

$$\mathbf{v} = \mathbf{v}' + \mathbf{u}r'/c, \qquad (34.4)$$

where **u** is some middle acceleration between the advanced acceleration u' and the observation acceleration u. To be able to carry out the calculations, we must have the same symbol for v in the nominator and denominator of **A**. Then, after having done the differentiations, we shall substitute v in all corrective terms by **v**' and in the non-corrective (or substantial) terms according to the relation (34.4). Then we shall do the same with the acceleration which will appear after taking time derivative from the velocity. As we shall see, the velocity will appear in the final result only in corrective terms and the acceleration only in substantial terms. Thus the substitution which we have to do in the final result will be

$$\mathbf{v} = \mathbf{v}', \qquad \mathbf{u} = \mathbf{u}' + \mathbf{w}'r'/c, \qquad (34.5)$$

where **w**' is the advanced super-acceleration of the charge.

Official physics asserts that the potentials which one has to use at the calculation of the electromagnetic field of an accelerated charge must be given by formulas (34.3) where v is to be substituted by **v**'. Such potentials, however, are neither advanced nor observation, as the pure advanced potentials will be

$$\Phi' = q/r', \qquad \mathbf{A}' = q\mathbf{v}'/cr', \qquad (34.6)$$

while the observation potentials

$$\Phi = q/r, \qquad \mathbf{A} = q\mathbf{v}/cr, \qquad (34.7)$$

if expressed through the advanced elements of motion, are to be written in the form (34.3) where v in the nominator of A is to be presented according to (34.4) through the advanced velocity and acceleration (as already said, v in the denominators of Φ and **A** is neither the advanced nor the observation velocity of the charge but some middle velocity). Thus official physics works[23] with some "hybrid" potentials which are neither pure advanced nor observation and for this reason it cannot obtain the radiation reaction intensity straightforwardly, as I do it in my theory considering **v** in the nominator of **A** as the observation velocity, so that Φ and **A** in (34.3) are the exact observation potentials (when assuming that light has a Marinov-aether character of propagation).

But why must we express the observation elements of motion in (34.3) - the charge-observer distance and the charge's velocity - through the advanced ones? The reason is not the hypothetical "propagation of interaction". I noticed already that as the quickest "information link" can be established by the help of light signals, one cannot calculate the intensities of a moving charge taking its position, velocity and acceleration at this very moment because there is no way to know them. At the reference point one can have information only about the advanced (or retarded) elements of motion.

There is, however, also another reason. As the radiated energy propagates with the velocity of light, then to calculate the radiated intensities at the reference point at the observation moment, one must operate with the advanced charge and current densities. Thus we are impelled to express the observation elements of motion in (34.3) by the advanced ones in order to obtain right values for the radiated intensities. The mechanics of the right calculation when radiation and potential intensities are to be separated becomes very transparent and clear in Sect. 37.

Let us now do the calculations.

In formulas (34.1) we must differentiate Φ and $\mathbf{A}$ with respect to the coordinates x,y,z of the reference point and the time of observation t. But in the relations (34.3) the potentials are given as function of t' and only through the relation

$$r' = c(t - t') \qquad (34.8)$$

as composite functions of t. Now I shall write several relations which will be then used for the calculation of the composite derivatives.

Having in mind the first relation (34.5), we write

$$\mathbf{v} \cong \mathbf{v}' = - \partial \mathbf{r}'/\partial t', \qquad (34.9)$$

where $\mathbf{r}'$ is the vector of the advanced distance pointing from the charge to the reference point.

Differentiating the equality $r'^2 = \mathbf{r}'^2$ with respect to t', we obtain

$$r' \frac{\partial r'}{\partial t'} = \mathbf{r}'.\frac{\partial \mathbf{r}'}{\partial t'} \qquad (34.10)$$

and using here (34.9), we find

$$\frac{\partial r'}{\partial t'} = - \mathbf{n}'.\mathbf{v}. \qquad (34.11)$$

Differentiating (34.8) with respect to t and considering r' as a direct function of t', we find

$$\frac{\partial r'}{\partial t'}\frac{\partial t'}{\partial t} = c(1 - \frac{\partial t'}{\partial t}); \qquad (34.12)$$

putting here (34.11), we obtain

$$\frac{\partial t'}{\partial t} = \frac{1}{1 - \mathbf{n}'.\mathbf{v}/c}. \qquad (34.13)$$

Similarly, differentiating relation (34.8) with respect to $\mathbf{r}$ and taking into account that t is the independent variable, we obtain

$$\frac{\partial r'}{\partial \mathbf{r}'}\frac{\partial \mathbf{r}'}{\partial \mathbf{r}} + \frac{\partial r'}{\partial t'}\frac{\partial t'}{\partial \mathbf{r}} = - c\,\frac{\partial t'}{\partial \mathbf{r}}; \qquad (34.14)$$

putting here (34.11), we obtain

$$\frac{\partial t'}{\partial \mathbf{r}} = - \frac{\mathbf{n}'}{c(1 - \mathbf{n}'.\mathbf{v}/c)}. \qquad (34.15)$$

Finally we find the following relation (which will be used only for the calculation of **B**)

$$\frac{\partial}{\partial \mathbf{r}}(r' - \frac{\mathbf{r}'.\mathbf{v}}{c}) = \frac{\partial}{\partial \mathbf{r}'}(r' - \frac{\mathbf{r}'.\mathbf{v}}{c}) + \frac{\partial}{\partial t'}(r' - \frac{\mathbf{r}'.\mathbf{v}}{c})\frac{\partial t'}{\partial \mathbf{r}} = \quad (34.16)$$

$$\mathbf{n}' - \frac{\mathbf{v}}{c} + (\mathbf{n}'.\mathbf{v} - \frac{v^2}{c} + \frac{\mathbf{r}'.\mathbf{u}}{c})\frac{\mathbf{n}'}{c(1 - \mathbf{n}'.\mathbf{v}/c)} = - \frac{\mathbf{v}}{c} + (c - \frac{v^2}{c} + \frac{\mathbf{r}'.\mathbf{u}}{c})\frac{\mathbf{n}'}{c(1 - \mathbf{n}'.\mathbf{v}/c)}.$$

Thus the electric intensity is to be calculated according to the formula (see (34.1))

$$\mathbf{E} = - \frac{\partial \Phi}{\partial \mathbf{r}} - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} = - \frac{\partial \Phi}{\partial \mathbf{r}'} - \frac{\partial \Phi}{\partial t'}\frac{\partial t'}{\partial \mathbf{r}} - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t'}\frac{\partial t'}{\partial t}. \quad (34.17)$$

If we substitute here the expressions (34.3) and take into account the relations (34.13) and (34.15), after some manipulations, the following final result can be obtained

$$\mathbf{E} = q\,\frac{1 - v^2/c^2}{(r' - \mathbf{r}'.\mathbf{v}/c)^3}\,(\mathbf{r}' - \frac{r'}{c}\,\mathbf{v}) + \frac{q\mathbf{r}'\times\{(\mathbf{r}' - r'\mathbf{v}/c)\times\mathbf{u}\}}{c^2(r' - \mathbf{r}'.\mathbf{v}/c)^3}, \quad (34.18)$$

where, according to (34.5), **v** is to be replaced by **v'**, as it appears only in corrective terms, and **u** is to be replaced by **u'** + **w'**r'/c, as it apppears in non-corrective terms.

One can easily check the equality of formulas (34.17) and (34.18) by reducing the first and the second to common denominators and by resolving all products to sums of single terms; then, after canceling mutually some terms in the nominator of formula (34.17), one sees that the remaining terms are equal to the terms in the nominator of formula (34.18).

Remembering the formula for rotation from a product of a vector and a scalar (see p. 6), we have to calculate the magnetic intensity according to the formula

$$\mathbf{B} = \mathrm{rot}\frac{q\mathbf{v}}{c(r' - \mathbf{r}'.\mathbf{v}/c)} = \frac{q}{c(r' - \mathbf{r}'.\mathbf{v}/c)}\,\mathrm{rot}\mathbf{v} - \frac{q}{c}\,\mathbf{v}\times\mathrm{grad}\frac{1}{r' - \mathbf{r}'.\mathbf{v}/c}. \quad (34.19)$$

Since we consider the velocity **v** as a function of **r** through the advanced time t', we shall have according to the rules for the differentiation of a composite function

$$\mathrm{rot}\mathbf{v}(t') = - \frac{\partial \mathbf{v}}{\partial t}\times\frac{\partial t'}{\partial \mathbf{r}}. \quad (34.20)$$

Substituting (34.15) into (34.20) and (34.20) into (34.19), we obtain

$$\mathbf{B} = \frac{q}{c^2(r' - \mathbf{r}'.\mathbf{v}/c)^2}\,\mathbf{u}\times\mathbf{r}' + \frac{q}{c(r' - \mathbf{r}'.\mathbf{v}/c)^2}\,\mathbf{v}\times\mathrm{grad}(r' - \mathbf{r}'.\mathbf{v}/c). \quad (34.21)$$

Putting here (34.16), we get

$$\mathbf{B} = \frac{q}{c^2(r' - \mathbf{r}'.\mathbf{v}/c)^3}\,\mathbf{r}'\times(-r'\mathbf{u} + \frac{\mathbf{r}'.\mathbf{v}}{c}\,\mathbf{u} - c\mathbf{v} + \frac{v^2}{c}\,\mathbf{v} - \frac{\mathbf{r}'.\mathbf{u}}{c}\,\mathbf{v}). \quad (34.22)$$

Forming the product $\mathbf{n}'\times\mathbf{E}$ (take $\mathbf{E}$ from (34.18)), we obtain an expression equal to the right side of (34.22) and, thus, we conclude

$$\mathbf{B} = \mathbf{n}'\times\mathbf{E}. \tag{34.23}$$

Now substituting $\mathbf{v}$ and $\mathbf{u}$ from (34.5), we can present $\mathbf{E}$ in a form where only advanced quantities are present

$$\mathbf{E} = q\,\frac{(1 - v'^2/c^2)(\mathbf{n}' - \mathbf{v}'/c)}{r'^2(1 - \mathbf{n}'.\mathbf{v}'/c)^3} + \frac{q}{c^2}\,\frac{\mathbf{n}'\times\{(\mathbf{n}' - \mathbf{v}'/c)\times\mathbf{u}'\}}{r'(1 - \mathbf{n}'.\mathbf{v}'/c)^3} + \frac{q}{c^3}\,\mathbf{n}'\times(\mathbf{n}'\times\mathbf{w}'). \tag{34.24}$$

In the last term depending on the super-acceleration we have not taken into account the factors which will give terms, where c will be in a power higher than 3 in the denominator, as such terms are negligibly small.

Substituting (34.24) into (34.23), we obtain the following expression for the magnetic intensity where only advanced quantities are present

$$\mathbf{B} = -\frac{q}{c}\,\frac{(1 - v'^2/c^2)\mathbf{n}'\times\mathbf{v}'}{r'^2(1 - \mathbf{n}'.\mathbf{v}'/c)^3} + \frac{q}{c^2}\,\frac{\mathbf{n}'\times[\mathbf{n}'\times\{(\mathbf{n}' - \mathbf{v}'/c)\times\mathbf{u}'\}]}{r'(1 - \mathbf{n}'.\mathbf{v}'/c)^3} - \frac{q}{c^3}\,\mathbf{n}'\times\mathbf{w}'. \tag{34.25}$$

34.2. CALCULATION WITH THE RETARDED ELEMENTS OF MOTION.

Entirely in the same way as in Sect. 34.1 we can calculate the electric and magnetic intensities produced by a charge moving with acceleration, if expressing the observation elements of motion through the retarded ones. These calculations are done in Ref. 5. Here I shall give only the final formulas which are analogous to formulas (34.24) and (34.25)

$$\mathbf{E} = q\,\frac{(1 - v''^2/c^2)(\mathbf{n}'' + \mathbf{v}''/c)}{r''^2(1 + \mathbf{n}''.\mathbf{v}''/c)^3} + \frac{q}{c^2}\,\frac{\mathbf{n}''\times\{(\mathbf{n}'' + \mathbf{v}''/c)\times\mathbf{u}''\}}{r''(1 + \mathbf{n}''.\mathbf{v}''/c)^3} - \frac{q}{c^3}\,\mathbf{n}''\times(\mathbf{n}''\times\mathbf{w}''), \tag{34.26}$$

$$\mathbf{B} = -\frac{q}{c}\,\frac{(1 - v''^2/c^2)\mathbf{n}''\times\mathbf{v}''}{r''^2(1 + \mathbf{n}''.\mathbf{v}''/c)^3} - \frac{q}{c^2}\,\frac{\mathbf{n}''\times[\mathbf{n}''\times\{(\mathbf{n}'' + \mathbf{v}''/c)\times\mathbf{u}''\}]}{r''(1 + \mathbf{n}''.\mathbf{v}''/c)^3} - \frac{q}{c^3}\,\mathbf{n}''\times\mathbf{w}'', \tag{34.27}$$

and the formulas for the observation potentials expressed through the retarded elements of motion, from which we proceed and which are analogical to formulas (34.3)

$$\phi = \frac{q}{r'' + \mathbf{r}''.\mathbf{v}/c}, \qquad \mathbf{A} = \frac{q\mathbf{v}}{c(r'' + \mathbf{r}''.\mathbf{v}/c)}. \tag{34.28}$$

34.3. INTERPRETATION OF THE OBTAINED RESULTS.

I shall use the formulas written with the advanced elements of motion.

The three terms in formulas (34.24) and (34.25) are called, respectively, POTENTIAL, RADIATION and RADIATION REACTION ELECTRIC and MAGNETIC INTENSITIES.

Replacing again the advanced velocity by the observation velocity (see (34.5)), the potential electric intensity can be written

$$\mathbf{E}_{pot} = q\,\frac{1 - v^2/c^2}{(r' - \mathbf{r}'.\mathbf{v}/c)^3}(\mathbf{r}' - \mathbf{v}r'/c), \tag{34.29}$$

Using fig. 1, we can write

$$r' - \mathbf{r}'.\mathbf{v}/c = r' - r'v\cos\theta'/c = \{r^2 - (r'v\sin\theta'/c)^2\}^{1/2}. \qquad (34.30)$$

According to the law of sines we have

$$r'/\sin(\pi - \theta) = r/\sin\theta', \qquad (34.31)$$

so that we can write (34.30) in the form

$$r' - \mathbf{r}'.\mathbf{v}/c = r(1 - v^2\sin^2\theta/c)^{1/2}. \qquad (34.32)$$

Substituting this into (34.29) and putting there $\mathbf{r} = \mathbf{r}' - \mathbf{v}r'/c$, we obtain

$$\mathbf{E}_{pot} = q\,\frac{1 - v^2/c^2}{(1 - v^2\sin^2\theta/c^2)^{3/2}}\,\frac{\mathbf{r}}{r^3} \cong q\,\frac{\mathbf{n}}{r^2}. \qquad (34.33)$$

In the same way we obtain for the potential magnetic intensity

$$\mathbf{B}_{pot} = \frac{q}{c}\,\frac{1 - v^2/c^2}{(1 - v^2\sin^2\theta/c^2)^{3/2}}\,\frac{\mathbf{v}\times\mathbf{r}}{r^3} \cong \frac{q}{c}\,\frac{\mathbf{v}\times\mathbf{n}}{r^2}. \qquad (34.34)$$

I consider the difference between the "exact" and "non-exact" values of the potential electric and magnetic intensities as due only to the aether-Marinov character of light propagation. Thus I hardly believe that this can be an effect which can be physically observed. Conventional physics accepts that the "field" of a rapidly moving charge concentrates to a plane perpendicular to its motion, as for $\theta \to \pi/2$ there is $(1 - v^2/c^2)(1 - v^2\sin^2\theta/c^2)^{3/2} \to \infty$ when $v \to c$. I think that the effect is only computational and that it cannot be observed. Of course, the last word has the experiment.

Thus the potential electric and magnetic intensities of an arbitrarily moving electric charge are determined by the distance from the charge to the reference point (being inversely proportional to the square of this distance) and (for **B**) by the velocity of the charge, both taken at the moment of observation. These intensities are exactly equal to the electromagnetic intensities which the charge will originate at the reference point if the velocity is constant.

The second terms on the right sides of (34.24) and (34.25)

$$\mathbf{E}_{rad} = \frac{q}{c^2}\,\frac{\mathbf{n}'\times\{(\mathbf{n}' - \mathbf{v}'/c)\times\mathbf{u}'\}}{r'(1 - \mathbf{n}'.\mathbf{v}/c)^3}, \qquad \mathbf{B}_{rad} = \mathbf{n}'\times\mathbf{E}_{rad} \qquad (34.35)$$

determine the electric and magnetic intensities which the energy radiated by the charge originates at the reference point and we call them radiation electric and magnetic intensities. As the radiated energy propagates in space with the velocity of light c, we do not have to express here the advanced elements by the observation elemenets. Here the "directional" effects are no more computational and they can easily be observed(5). The radiation electric and magnetic intensities are determined by the distance from the charge to the reference point (being inversely proportional to this distance) and by the acceleration of the charge taken at the advanced moment. Thus a charge moving with a constant velocity does not originate radiation intensities.

The third terms on the right sides of (34.24) and (34.25)

$$E_{rea} = \frac{q}{c^3} \mathbf{n}'\times(\mathbf{n}'\times\mathbf{w}'), \qquad B_{rea} = -\frac{q}{c^3} \mathbf{n}'\times\mathbf{w}' = \mathbf{n}'\times E_{rea} \qquad (34.36)$$

determine the electric and magnetic intensities acting on the radiating charge itself as a reaction to the photon radiation diminishing its velocity and consequently its kinetic energy with a quantity exactly equal to the quantity of energy radiated in the form of photons.

The radiation intensities are those which appear at the reference point when the radiated photons cross this point; if there are electric charges at the reference point, they will come into motion "absorbing" the radiated energy. The radiation reaction intensities act on the radiating charge itself. For this reason I call the intensities (34.36) electric and magnetic intensities of radiation reaction.

The electric and magnetic intensities of radiation reaction do not depend on the distance between charge and reference point and are determined by the charge's super-acceleration at the advanced moment, which, of course, can be taken equal to the super-acceleration at the observation moment.

Thus we see that only the potential and radiation intensities have a character of field quantities, because when position, velocity and acceleration of the charge are given, these intensities are determined in all points of space, the former "momentarily", the latter with a time delay r'/c. The radiation reaction intensities are determined only for the space point where the radiating charge is located and act only on this charge.

One may wonder that such precised, detailed and complicated information can be obtained with some simple mathematics from the extremely simple initial equations (34.3) and (34.1), so that here we have to admire the Divinity for His superb perfectness and amazing abilities.

Entirely in the same way, we can establish that the first terms in formulas (34.26) and (34.27) give, respectively, the potential electric and magnetic intensities (34.33) and (34.34). Thus we conclude that the calculation of the potential electric and magnetic intensities with the help of the advanced elements of motion as well as with the retarded elements of motion leads exactly to the same results.

Let us now compare the second and third terms in formulas (34.24), (34.25) and in formulas (34.26), (34.27). If we assume that the advanced elements of motion do not differ too much from the retarded ones, i.e., if we assume

$$\mathbf{r}' = \mathbf{r}'' = \mathbf{r}, \qquad \mathbf{v}' = \mathbf{v}'' = \mathbf{v}, \qquad u' = u'' = u, \qquad \mathbf{w}' = \mathbf{w}'' = \mathbf{w}, \qquad (34.37)$$

then the electric intensity given by formulas (34.24) and (34.26) and the magnetic intensity given by formulas (34.25) and (34.27) can be written as follows

$$\mathbf{E} = \mathbf{E}_{pot} + \mathbf{E}_{rad} + \mathbf{E}_{rea} = q\frac{\mathbf{n}}{r^2} + q\frac{\mathbf{n}\times(\mathbf{n}\times\mathbf{u})}{c^2 r} \pm q\frac{\mathbf{n}\times(\mathbf{n}\times\mathbf{w})}{c^3},$$

$$\mathbf{B} = \mathbf{B}_{pot} + \mathbf{B}_{rad} + \mathbf{B}_{res} = -q\frac{\mathbf{n}\times\mathbf{v}}{c\, r^2} \mp q\frac{\mathbf{n}\times\mathbf{u}}{c^2 r} - q\frac{\mathbf{n}\times\mathbf{w}}{c^3}, \qquad (34.38)$$

where the upper signs are obtained when the calculation is carried out by the help of the advanced elements of motion, and the lower signs are obtained when the calculation is carried out by the help of the retarded elements of motion.

As said above, the potential intensities are the same when calculated with the advanced and with the retarded elements of motion.

The electric intensity of radiation $\mathbf{E}_{rad}$ is the same when calculated with the advanced and with the retarded elements of motion. However the magnetic intensity of radiation $\mathbf{B}_{rad}$ is obtained with opposite sign if the retarded elements are used. Since we relate the intensities of radiation with the density of the energy flux (see Sect. 14)

$$\mathbf{I} = (c/4\pi)\mathbf{E}_{rad}\times\mathbf{B}_{rad}, \qquad (34.39)$$

we see that the electric and magnetic radiation intensities calculated with the advanced elements of motion give an energy flux density directed from the charge to the reference point

$$(4\pi/c)\mathbf{I}' = \mathbf{E}'_{rad}\times\mathbf{B}'_{rad} = -\frac{q^2}{c^4 r^2}\{\mathbf{n}\times(\mathbf{n}\times\mathbf{u})\times(\mathbf{n}\times\mathbf{u}) = -\frac{q^2}{c^4 r^2}\{(\mathbf{n}.\mathbf{u})\mathbf{n} - (\mathbf{n}.\mathbf{n})\mathbf{u}\}\times(\mathbf{n}\times\mathbf{u}) =$$

$$-\frac{q^2}{c^4 r^2}\{(\mathbf{n}.\mathbf{u})\mathbf{n}\times(\mathbf{n}\times\mathbf{u}) - \mathbf{u}\times(\mathbf{n}\times\mathbf{u})\} = -\frac{q^2}{c^4 r^2}\{(\mathbf{n}.\mathbf{u})^2\mathbf{n} - u^2\mathbf{n}\} = \frac{q^2}{c^4 r^2}\{u^2 - (\mathbf{n}.\mathbf{u})^2\}\mathbf{n}, \qquad (34.40)$$

while the electric and magnetic intensities of radiation calculated with the retarded elements of motion give an energy flux density directed from the reference point to the charge

$$(4\pi/c)\mathbf{I}'' = \mathbf{E}''_{rad}\times\mathbf{B}''_{rad} = -\frac{q^2}{c^4 r^2}\{u^2 - (\mathbf{n}.\mathbf{u})^2\}\mathbf{n}. \qquad (34.41)$$

As $u^2 - (\mathbf{n}.\mathbf{u})^2 \geq 0$, the flux (34.40) corresponds to the real electromagnetic wave radiated in the direction $\mathbf{n}$, while the flux (34.41) corresponds to a wave propagating in the direction $-\mathbf{n}$. This second wave is fictitious, as it must exist if time has the property "reversibility". Thus only the calculation with the advanced elements of motion corresponds to the real course of time (from the past to the future); the calculation with the retarded elements of motion corresponds to the negative course of time (from the future to the past).

The intensities of radiation reaction do not depend on the distance between the charge and the reference point, and, thus, they have mathematical sense also for the point where the charge itself is placed. So we are impelled to make the conclusion that the electric and magnetic intensities of radiation reaction act on the radiating charge itself. Here we cannot speak about advanced and retarded moments, as both these moments coincide with the observation moment.

However, as formulas (34.38) show, the intensities E_{rea} and B_{rea} depend on the angle between the super-acceleration and the line connecting the charge with the reference point. Since the reference point for the radiation reaction is the radiating charge itself, we have to eliminate such an angular dependence by averaging over all directions.

The averaging is to be performed in the following way: We plot the vectors of the intensities E_{rea} obtained when the reference point covers densily a whole sphere around the charge, so that the angle between **n** and **w** takes all possible values. Now if we add geometrically all these vectors E_{rea_i}, $i = 1,2,\ldots..N$, whene $N \to \infty$, and if we divide the resultant vector by the number N, we shall find the average value (we write the intensity of radiation reaction calculated with the advanced elements of motion)

$$\overline{\mathbf{E}_{rea}} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{E}_{rea_i} = \frac{1}{N}\sum_{i=1}^{N} q\mathbf{n}_i\times(\mathbf{n}_i\times\mathbf{w})/c^3. \tag{34.42}$$

Multiplying both sides of this equation by 4π, we get

$$4\pi\overline{\mathbf{E}_{rea}} = \sum_{i=1}^{N}\mathbf{E}_{rea_i}\frac{4\pi}{N} = \int_{4\pi}\mathbf{E}_{rea}d\Omega, \tag{34.43}$$

by making the transition $N \to \infty$, and thus

$$\overline{\mathbf{E}_{rea}} = \frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi} q\,\frac{\mathbf{n}\times(\mathbf{n}\times\mathbf{w})}{c^3}\sin\theta\, d\theta\, d\phi = \frac{q}{4\pi c^3}\int_0^{\pi}\int_0^{2\pi}\{(\mathbf{n}.\mathbf{w})\mathbf{n} - \mathbf{w}\}\sin\theta\, d\theta\, d\phi, \tag{34.44}$$

where $n_x = \sin\theta\cos\phi$, $n_y = \sin\theta\sin\phi$, $n_z = \cos\theta$, θ and ϕ being the zenith and azimuth angles of a spherical frame of reference with origin at the charge.

Thus formula (34.44) can be written

$$\begin{aligned}
\overline{\mathbf{E}_{rea}} &= \frac{q}{4\pi c^3}\int_0^{\pi}\int_0^{2\pi}\{(w_x\sin\theta\cos\phi + w_y\sin\theta\sin\phi + w_z\cos\theta)(\sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \\
&\qquad \cos\theta\,\hat{\mathbf{z}} - \mathbf{w}\}\sin\theta\, d\theta\, d\phi = \\
&\frac{q}{4\pi c^3}\, w_x\hat{\mathbf{x}}\int_0^{\pi}\int_0^{2\pi}\sin^3\theta\cos^2\phi\, d\theta\, d\phi + w_y\hat{\mathbf{y}}\int_0^{\pi}\int_0^{2\pi}\sin^3\theta\sin^2\phi\, d\theta\, d\phi + \\
&\qquad w_z\hat{\mathbf{z}}\int_0^{\pi}\int_0^{2\pi}\cos^2\theta\sin\theta\, d\theta\, d\phi - \mathbf{w}\int_0^{\pi}\int_0^{2\pi}\sin\theta\, d\theta\, d\phi\} = \\
&\frac{q}{4c^3}\{w_x\hat{\mathbf{x}}\int_0^{\pi}\sin^3\theta\, d\theta + w_y\hat{\mathbf{y}}\int_0^{\pi}\sin^3\theta\, d\theta + w_z\hat{\mathbf{z}}\int_0^{\pi}2\cos^2\theta\sin\theta\, d\theta - \mathbf{w}\int_0^{\pi}2\sin\theta\, d\theta\} = \\
&\frac{q}{4c^3}\left(\frac{4}{3}w_x\hat{\mathbf{x}} + \frac{4}{3}w_y\hat{\mathbf{y}} + \frac{4}{3}w_z\hat{\mathbf{z}} - 4\mathbf{w}\right) = \frac{q}{4c^3}\left(\frac{4}{3}\mathbf{w} - 4\mathbf{w}\right) = -\frac{2q}{3c^3}\mathbf{w}.
\end{aligned} \tag{34.45}$$

The magnetic intensities of radiation reaction are the same when calculated with the help of the advanced and retarded elements of motion. But the averaging of the magnetic intensity of radiation reaction over all angles gives zero. Indeed,

$$\overline{B_{rea}} = \frac{1}{4\pi}\int_{4\pi} B_{rea} d\Omega = - \frac{q}{4\pi c^3}\int_0^\pi\int_0^{2\pi} \mathbf{n}\times\mathbf{w} \sin\theta\, d\theta\, d\phi =$$

$$- \frac{q}{4\pi c^3}\int_0^\pi\int_0^{2\pi} \{(w_z \sin\theta \sin\phi - w_y \cos\theta)\hat{x} + (w_x \cos\theta - w_z \sin\theta \cos\phi)\hat{y} +$$

$$(w_y \sin\theta \cos\phi - w_x \sin\theta \sin\phi)\hat{z}\} \sin\theta\, d\theta\, d\phi = 0. \qquad (34.46)$$

Thus formulas (34.38) are to be written in the form

$$\mathbf{E} = \mathbf{E}_{pot} + \mathbf{E}_{rad} + \mathbf{E}_{rea} = q\,\frac{\mathbf{n}}{r^2} + q\,\frac{\mathbf{n}\times(\mathbf{n}\times\mathbf{u})}{c^2 r} - \frac{2q}{3c^2}\mathbf{w},$$

$$\mathbf{B} = \mathbf{B}_{pot} + \mathbf{B}_{rad} = - q\,\frac{\mathbf{n}\times\mathbf{v}}{cr^2} - q\,\frac{\mathbf{n}\times\mathbf{u}}{c^2 r}, \qquad (34.47)$$

where we have taken these signs which correspond to the calculation with the advanced elements of motion.

35. ELECTROMAGNETIC POTENTIALS OF PERIODIC SYSTEMS

Let us suppose that the charge and current densities of the considered system of electric charges are simple periodic (i.e., monoperiodic, or trigonometric) functions of time

$$Q = Q_{max}\cos(\frac{2\pi}{T}t + \alpha), \quad \mathbf{J} = \mathbf{J}_{max}\cos(\frac{2\pi}{T}t + \alpha), \qquad (35.1)$$

where Q_{max} and $\mathbf{J}_{max}$ are the amplitudes of the charge and current densities and represent their values for times $t = nT - (\alpha/2\pi)T$, where n is an integer.

The quantity T is the PERIOD of the charge and current fluctuations; this is the time after whose expiration the charge and current densities obtain again the same values. The argument $2\pi t/T + \alpha$ of the trigonometric function is the PHASE and the quantity α is the initial phase which usually, when considering the charge and current densities at a given space point only, can be taken equal to zero. The quantity $\omega = 2\pi/T$ is called (CIRCULAR) FREQUANCY and the quantity $k = \omega/c = 2\pi/cT$ is called (CIRCULAR) WAVE NUMBER. Such an electromagnetic SYSTEM is called MONOPERIODIC.

It is mathematically more convenient to write the real trigonometric relations as complex exponential relations. Thus we can present the expressions (35.1) in the form

$$Q = \mathrm{Re}\{Q_{max}e^{i(\omega t+\alpha)}\} = \mathrm{Re}\{Q_{max}e^{-i(\omega t+\alpha)}\},$$

$$\mathbf{J} = \mathrm{Re}\{\mathbf{J}_{max}e^{i(\omega t+\alpha)}\} = \mathrm{Re}\{\mathbf{J}_{max}e^{-i(\omega t+\alpha)}\}, \qquad (35.2)$$

where Re{ } means that we must take only the real part of the complex expression in the braces. The real parts of both expressions (35.2) are equal but usually the second forms are used, i.e., those with the negative exponents.

If we introduce the notations

$$Q_\omega = Q_{max} e^{-i\alpha}, \qquad \mathbf{J}_\omega = \mathbf{J}_{max} e^{-i\alpha}, \qquad (35.3)$$

we can write (35.2), by omitting the sign Re{ }, in the form

$$Q = Q_\omega e^{-i\omega t}, \qquad \mathbf{J} = \mathbf{J}_\omega e^{-i\omega t}, \qquad (35.4)$$

where the new amplitudes Q_ω, $\mathbf{J}_\omega$ must be considered as complex numbers which become real only under the condition $\alpha = 0$. The complex forms (35.2) are called SHORT EXPONENTIAL FORMS and the complex forms (35.4) are called LAPIDARY EXPONENTIAL FORMS. The LONG EXPONENTIAL FORMS are the following

$$Q = (1/2)\{Q_\omega e^{-i\omega t} + Q_\omega^* e^{i\omega t}\}, \qquad \mathbf{J} = (1/2)\{\mathbf{J}_\omega e^{-i\omega t} + \mathbf{J}_\omega^* e^{i\omega t}\}, \qquad (35.5)$$

where Q_ω^*, $\mathbf{J}_\omega^*$ are the quantities complex conjugated to Q_ω, $\mathbf{J}_\omega$.

The use of the complex exponential forms turns out to be very convenient when we perform linear operations (say, adding, differentiation, integration) over the trigonometric functions. By using the complex exponential forms, all linear operations are to be applied not to trigonometric but to much simpler exponential expressions. However, when we have to perform non-linear operations (say, multiplication), we have always to use the long exponential forms.

Let us find the electric and magnetic potentials originated by a monoperiodic system at an arbitrary reference point.

Following the concept that the potential electric and magnetic intensities appear "momentarily" in whole space, while the radiated intensities propagate with the velocity c, we shall bear in mind the following rules when calculating the intensities proceeding from the potentials:

1) When we calculate the potential intensities, we have to use the observation potentials (refer to formula (34.7)).

2) When we calculate the radiation intensities, we have to use the advanced potentials (refer to formula (34.6).

3) When we calculate both the potential and radiation intensities, we have to use the advanced potentials (see formulas (10.3))

$$\Phi = \int_V \frac{Q(t - R/c)}{R} dV, \qquad \mathbf{A} = \int_V \frac{\mathbf{J}(t - R/c)}{R} dV, \qquad (35.6)$$

where R is the distance to the elementary volume dV, but in the final result we have to put $c = \infty$ in all non-radiation intensities if this c appears as a result of manipulation with advanced time. The execution of this program will become clear in Sect. Sect. 37.

Thus if the charge and current densities at every elementary volume of the considered system are simple periodic functions of time, with equal periods of fluctuations, the electric and magnetic potentials will be also simple periodic functions of time with the same period and by putting (35.4) into (35.6) we obtain

$$\Phi(t) = \Phi_\omega e^{-i\omega t} = \int_V \frac{Q_\omega}{R} e^{-i(\omega t - kR)} dV, \qquad A(t) = A_\omega e^{-i\omega t} = \int_V \frac{J_\omega}{R} e^{-i(\omega t - kR)} dV,$$

where

$$\Phi_\omega = \int_V \frac{Q_\omega}{R} e^{ikR} dV, \qquad A_\omega = \int_V \frac{J_\omega}{R} e^{ikR} dV \qquad (35.8)$$

are the complex amplitudes of the advanced electric and magnetic potentials.

Let us now suppose that the charge and current densities are periodic,but not trigonometric, functions of time. As it is known, any periodic function can be presented as a Fourier series, i.e., as a superposition of trigonometric functions with different periods. We shall call such SYSTEMS POLYPERIODIC and their potentials will be superposition of potentials of monoperiodic systems.

If the charge and current densities are arbitrary functions of time, then, as it is known, they can be presented by a Fourier integral as a superposition of monoperiodic functions and such will be also the potentials. We call such systems APERIODIC.

36. THE POTENTIALS AT LARGE DISTANCES FROM THE GENERATING SYSTEM

Let us consider the potentials generated by an electromagnetic system of arbitrarily moving charges at large distances from the system, that is at distances which are large compared with the dimensions of the system.

We choose (fig. 16) the origin O of the reference frame somewhere in the interior of the system of charges using the following notations: the radius vector of the reference point P is denoted by **r** and the unit vector along it by **n**; the radius vector of the charges in the differential volume dV around point Q (where the charge and current densities are Q(t) and J(t), respectively) is denoted by **r**'; the radius vector from the the volume dV to the reference point P is denoted by **R**.

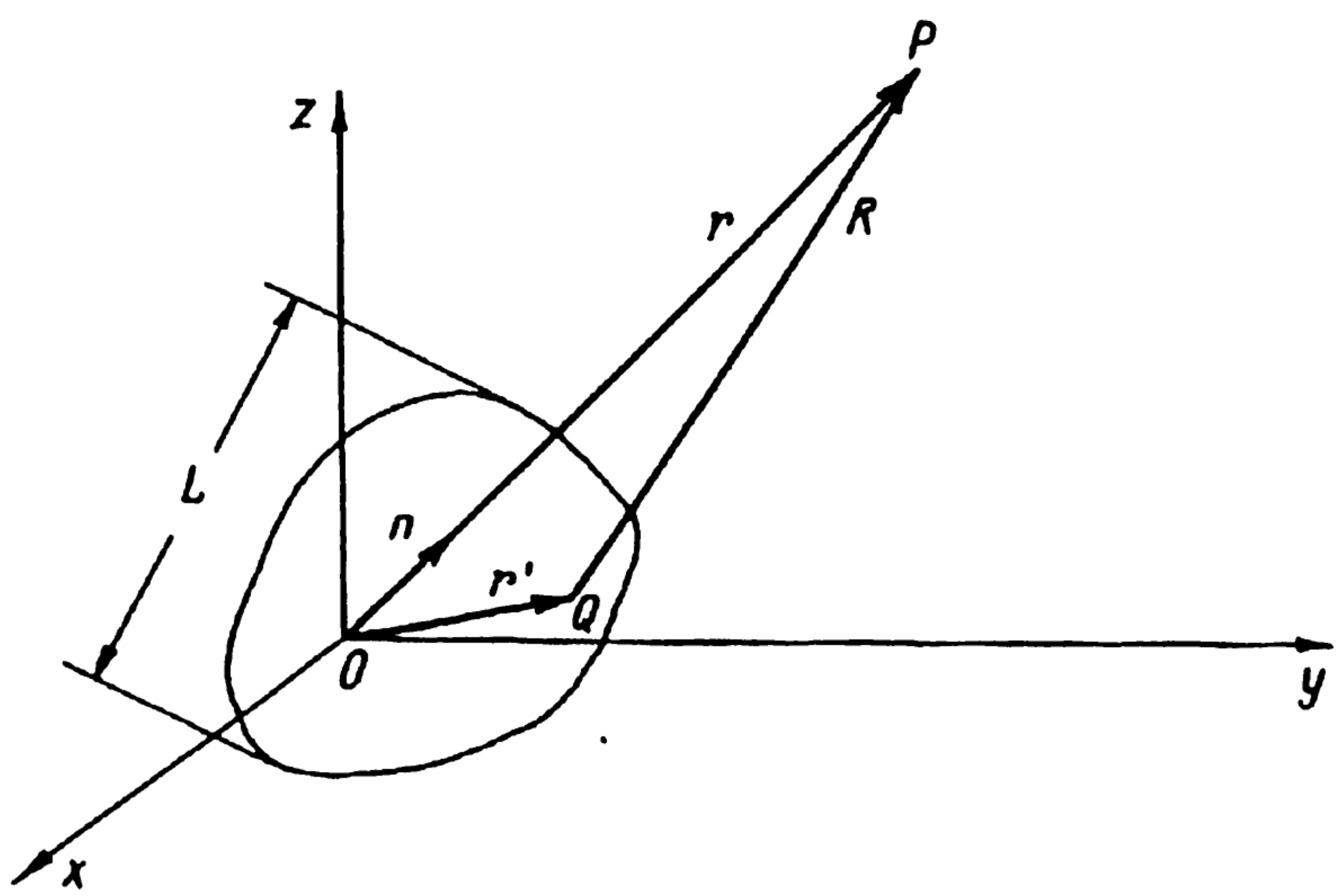

Fig. 16. Electromagnetic system and a far lying reference point.

Denoting by L the largest dimension of the system, we shall assume

$$r \gg L, \tag{36.1}$$

and therefore

$$r \gg r'. \tag{36.2}$$

From fig. 16 we have $\mathbf{R} = \mathbf{r} - \mathbf{r}'$, and thus we can write approximately

$$R = |\mathbf{r} - \mathbf{r}'| \cong (r^2 - 2\mathbf{r}.\mathbf{r}')^{1/2} = r(1 - 2\mathbf{n}.\mathbf{r}'/r)^{1/2} \cong r - \mathbf{n}.\mathbf{r}', \tag{36.3}$$

and with larger inaccuracy

$$R \cong r. \tag{36.4}$$

In addition to the condition (36.1) we shall sometimes assume also that the shortest period of oscillation T of the charge and current densities at the different elementary volumes of the system is much larger than the time in which light covers the largest dimension of the system, i.e.,

$$T \gg L/c. \tag{36.5}$$

Let us now consider the advanced magnetic potential of a monoperiodic system. Substituting (36.3) into the second formula (35.8), we shall have at this approximation

$$\mathbf{A}_\omega = \frac{1}{c}\int_V \frac{\mathbf{J}_\omega}{r - \mathbf{n}.\mathbf{r}'} e^{ik(r - \mathbf{n}.\mathbf{r}')} dV. \tag{36.6}$$

Taking into account assumption (36.2), we can neglect $\mathbf{n}.\mathbf{r}'$ with respect to r in the denominator. However, this condition <u>is not enough</u> to make the same neglection in the exponent of the nominator. Indeed, we have

$$\mathrm{Re}\{e^{ik(r - \mathbf{n}.\mathbf{r}')}\} = \cos\{\frac{2\pi}{cT}(r - \mathbf{n}.\mathbf{r}')\} = \cos[2\pi\{\frac{r}{cT} - \frac{r'}{cT}\cos(\mathbf{n}.\mathbf{r}')\}]. \tag{36.7}$$

Thus we can neglect in this expression $(r'/cT)\cos(\mathbf{n}.\mathbf{r}')$ only if $r'/cT < L/cT \ll 1$, i.e., if also condition (36.5) is fulfilled.

Thus assuming that only condition (36.1) is fulfilled but condition (36.5) is not, we can write (36.6) in the form

$$\mathbf{A}_\omega = \frac{e^{ikR}}{cr}\int_V \mathbf{J}_\omega e^{-\mathbf{n}.\mathbf{r}'} dV. \tag{36.8}$$

Assuming that both conditions (36.1) and (36.5) are fulfilled, we can write (36.6) in the form

$$\mathbf{A}_\omega = \frac{e^{ikr}}{cr}\int_V \mathbf{J}_\omega dV. \tag{36.9}$$

These results can be applied to the first formula (35.8) and then to the electromagnetic potentials of polyperiodic and aperiodic systems.

Let us consider now the advanced magnetic potential of an arbitrary system written in the general form (35.6). Substituting (36.3) into (35.6), we shall have

$$\mathbf{A} = \frac{1}{c}\int_V \frac{\mathbf{J}(t - r/c + \mathbf{n}.\mathbf{r}'/c)}{r - \mathbf{n}.\mathbf{r}'}\, dV. \qquad (36.10)$$

Assuming that only condition (36.1) is fulfilled but condition (36.5) is not, we can write

$$\mathbf{A} = \frac{1}{cr}\int_V \mathbf{J}(t' + \mathbf{n}.\mathbf{r}'/c)dV, \qquad (36.11)$$

where $t' = t - r/c$ is the common advanced moment for the whole system, i.e., the advanced moment taken with respect to the frame's origin.

Expanding the integral in (36.11) as a power series of the small quantity $\mathbf{n}.\mathbf{r}'/c$, we obtain

$$\mathbf{A} = \mathbf{A}^{(o)} + \mathbf{A}^{(1)} + \ldots\ldots = \frac{1}{cr}\int_V \mathbf{J}(t')dV + \frac{1}{c^2 r}\int_V (\mathbf{n}.\mathbf{r}')\frac{d\mathbf{J}(t')}{dt'} + \ldots\ldots . \qquad (36.12)$$

Since $\mathbf{n}$ is a constant unit vector and the vectors $\mathbf{r}'$ are integration variables which do not depend on time, we can write, taking into account that $\mathbf{J}dV$ is equal to the sum of the charges in the volume dV multiplied by their velocities

$$\mathbf{A} = \frac{1}{cr}\sum_{i=1}^{n} q_i \mathbf{v}_i(t') + \frac{1}{c^2 r}\frac{d}{dt'}\sum_{i=1}^{n} q_i(\mathbf{n}.\mathbf{r}_i')\mathbf{v}_i(t') + \ldots\ldots\ldots . \qquad (36.13)$$

In zero approximation we have

$$\mathbf{A}^{(o)} = \frac{1}{cr}\sum_{i=1}^{n} q_i \mathbf{v}_i = \frac{1}{cr}\frac{d}{dt'}\sum_{i=1}^{n} q_i \mathbf{r}' = \frac{\dot{\mathbf{d}}}{cr}, \qquad (36.14)$$

where $\mathbf{d}$ is the advanced dipole moment of the system, and the point over the symbol signifies that time derivative is taken from this quantity. We remind that the elements of motion on the right side of the last formulas are taken at the common advanced moment.

37. POTENTIAL FIELD AND RADIATION FIELD

We established in Sect. 34 that the intensity field of an arbitrarily moving electric charge consists of two parts - potential part and radiation part. As formulas (34.38) show, the potential electric and magnetic intensities are inversely proportional to the second power of the distance from the charge producing them, while the radiation electric and magnetic intensities are inversely proportional to the first power of this distance. Then we established that the potential electromagnetic intensities "appear", as the potentials, instantly in whole space, i.e., they are immaterial, while the radiation electromagnetic intensities "propagate" with the velocity of light from the charge producing them to infinity; thus we have identified the radiation field of the charge by the photons emitted by it.

As the field of a system of arbitrarily moving charges represents a superposition of the fields of anyone of these charges, the common intensity field of the whole system will also consist of a potential part and a radiation part.

Let us now find the field of a system of charges at large distance from it. As mentioned in Sect. 35, for the calculation of the potential and radiation intensities we use the advanced potentials but then in all non-radiation intensity terms we have to put $c = \infty$ everywhere where this "c" appears as a result of manipulation with advanced time; non-radiation terms are all those which are not inversely proportional to the first power of the distance from the system to the reference point. The essence of this program will become clear in this section.

For simplicity sake, we shall make a calculation for the potentials taken in zero approximation. Thus the advanced magnetic potential will be given by formula (36.14). The advanced electric potential can be calculated by substituting (36.14) into the equation of potential connection (8.8)

$$\mathrm{div}(\dot{\mathbf{d}}/cr) = -(1/c)\partial\Phi/\partial t. \qquad (37.1)$$

After integration we can determine the electric potential

$$\Phi = -\,\mathrm{div}(\mathbf{d}/r) + \mathrm{Const}, \qquad (37.2)$$

where the constant of integration must have the form

$$\mathrm{Const} = \frac{1}{r}\sum_{i=1}^{n} q_i, \qquad (37.3)$$

because if we put the dipole moment equal to zero, we shall have, at the assumption (36.1),

$$\Phi = \frac{1}{r}\sum_{i=1}^{n} q_i, \qquad (37.4)$$

where n is the number of the charges in the system.

Let us assume that the sum of all charges in the system is zero. Then the advanced electric potential will have the form (37.2) with Const = 0. Putting this and (36.14) into the fundamental definition equalities (34.1), we obtain the following expressions for the electric and magnetic intensities

$$\mathbf{E} = \mathrm{grad}(\mathrm{div}\frac{\mathbf{d}}{r}) - \frac{1}{c^2}\frac{\ddot{\mathbf{d}}}{r}, \quad \mathbf{B} = \frac{1}{c}\,\mathrm{rot}\frac{\dot{\mathbf{d}}}{r}. \qquad (37.5)$$

Now I shall calculate the monoperiodic amplitudes of the electric and magnetic intensities, assuming that the charge densities are monoperiodic functions of time; if they are polyperiodic or aperiodic functions of time, then we shall assume that a suitable expansion in a Fouruer series or Fourier integral is performed.

The resultant advanced dipole moment of the system can be presented as a superposition of the advanced monoperiodic moments of the form

$$\mathrm{d}(t') = \mathrm{d}_\omega e^{-i\omega t'} = \mathrm{d}_\omega e^{-i\omega(t - r/c)} = \mathbf{d}_\omega e^{-i\omega t + ik}. \qquad (37.6)$$

We see that the velocity "c" which figures in the advanced time is included in the wave number k; hence in all non-radiation intensity terms of the final reslt we have to put k = 0.

The electric and magnetic intensities produced by this monoperiodic dipole moment will also be periodic functions with the same frequency

$$E(t) = E_\omega e^{-i\omega t}, \qquad B(t) = B_\omega e^{-i\omega t}. \qquad (37.7)$$

Substituting (37.6) and (37.7) into the first equation (37.5) and dividing the equation obtained by the common factor exp(-iωt), we obtain for the monoperiodic amplitude of the electric intensity with frequency ω the following expression

$$\mathbf{E}_\omega = \text{grad}\{\text{div}(\frac{e^{ikr}}{r}\mathbf{d}_\omega)\} + \frac{\omega^2}{c^2}\frac{e^{ikr}}{r}\mathbf{d}_\omega = \text{grad}(\mathbf{d}_\omega.\text{grad}\frac{e^{ikr}}{r}) + \frac{k^2}{r}e^{ikr}\mathbf{d}_\omega =$$

$$(\mathbf{d}_\omega.\text{grad})\text{grad}\frac{e^{ikr}}{r} + \frac{k^2}{r}e^{ikr}\mathbf{d}_\omega = (\mathbf{d}_\omega.\text{grad})\{(\frac{ik}{r^2} - \frac{1}{r^3})e^{ikr}\mathbf{r}\} + \frac{k^2}{r}e^{ikr}\mathbf{d}_\omega =$$

$$\{\mathbf{d}_\omega.(-\frac{2ik}{r^3} + \frac{3}{r^4} - \frac{k^2}{r^2} - \frac{ik}{r^3})e^{ikr}\mathbf{n}\}\mathbf{r} + (\frac{ik}{r^2} - \frac{1}{r^3})e^{ikr}\mathbf{d}_\omega + \frac{k^2}{r}e^{ikr}\mathbf{d}_\omega =$$

$$(-\frac{k^2}{r} - \frac{3ik}{r^2} + \frac{3}{r^3})e^{ikr}(\mathbf{d}_\omega.\mathbf{n})\mathbf{n} + (\frac{ik}{r^2} - \frac{1}{r^3})e^{ikr}\mathbf{d}_\omega + \frac{k^2}{r}e^{ikr}\mathbf{d}_\omega = \qquad (37.8)$$

$$\frac{k^2}{r}e^{ikr}\{\mathbf{d}_\omega - (\mathbf{d}_\omega.\mathbf{n})\mathbf{n}\} - \frac{ik}{r^2}e^{ikr}\{3(\mathbf{d}_\omega.\mathbf{n})\mathbf{n} - \mathbf{d}_\omega\} + \frac{1}{r^3}e^{ikr}\{3(\mathbf{d}_\omega.\mathbf{n})\mathbf{n} - \mathbf{d}_\omega\}.$$

The amplitude of the radiation electric intensity is the one which is inversely proportional to the first power of r; thus we can write

$$E_{\omega rad} = \frac{k^2}{r}e^{ikr}\,\mathbf{n}\times(\mathbf{d}_\omega\times\mathbf{n}). \qquad (37.9)$$

In all other terms we have to put k = 0 and these terms which remain will represent the amplitude of the potential electric intensity. Thus we shall have

$$E_{\omega pot} = \frac{1}{r^3}\{3(\mathbf{d}_\omega.\mathbf{n})\mathbf{n} - \mathbf{d}_\omega\}. \qquad (37.10)$$

I showed (see (31.12)) that this is the electric intensity generated by a <u>static</u> electric system with a total charge equal to zero and dipole moment (31.6) different from zero. The difference from the static system is only this that in the general dynamic monoperiodic case the potential electric intensity, according to formula (37.7) is a monoperiodic function of time.

The second term on the right side of (37.8) appears only as a result of the computation and when putting k = 0 disappears, i.e., it has <u>no</u> physical meaning.

Which are the errors of conventional physics which assumes that the interaction "propagates" with the velocity c? First it has to consider the second term on the right side of (37.8) as a real electric intensity. However nobody has measured such an intensity. Second, conventional physics considers the third term on the right side of (37.8) together with the factor e^{ikr}, i.e., it assumes that the potential electric intensity of a monoperiodic system has a "wave character". It is extremely easy to show experimentally that this assertion is not true, as I shall show beneath.

Let us now see which are the radiation and potential magnetic intensities of a system with monoperiodic dipole moment different from zero. Substituting (37.6) and (37.7) into the second equation (37.5) and divinding the equation obtained by the common factor $\exp(-i\omega t)$, we obtain for the monoperiodic amplitude of the magnetic intensity with frequency ω the following expression

$$\mathbf{B}_\omega = - i \frac{\omega}{c} \mathrm{rot}(\frac{e^{ikr}}{r} \mathbf{d}_\omega) = i \frac{\omega}{c} \mathbf{d}_\omega \times \mathrm{grad} \frac{e^{ikr}}{r} = i \frac{\omega}{c} \mathbf{d}_\omega \times \{(\frac{ik}{r} - \frac{1}{r^2}) e^{ikr} \mathbf{n}\} =$$

$$- \frac{k^2}{r} e^{ikr} \mathbf{d}_\omega \times \mathbf{n} - \frac{i\omega}{cr^2} e^{ikr} \mathbf{d}_\omega \times \mathbf{n}. \qquad (37.11)$$

The amplitude of the radiation magnetic intensity is the one which is inversely proportional to the first power of r; thus we can write

$$\mathbf{B}_{\omega rad} = \frac{k^2}{r} e^{ikr} \mathbf{n} \times \mathbf{d}_\omega. \qquad (37.12)$$

In the other term representing the amplitude of the potential magnetic intensity we have to put $k = 0$; so we obtain

$$\mathbf{B}_{\omega pot} = \frac{i\omega}{cr^2} \mathbf{n} \times \mathbf{d}_\omega. \qquad (37.13)$$

Having in mind (37.7) and (37.13), we can write the time depending potential magnetic intensity corresponding to the frequency ω in the form

$$\mathbf{B}_{pot}(t) = \frac{i\omega}{cr^2} \mathbf{n} \times \mathbf{d}_\omega e^{-i\omega t} = - \frac{\mathbf{n}}{cr^2} \times \frac{d}{dt} (\mathbf{d}_\omega e^{-i\omega t}) = - \frac{\mathbf{n}}{cr^2} \times \dot{\mathbf{d}}(t). \qquad (37.14)$$

Using now formula (36.14), we get

$$\mathbf{B}_{pot}(t) = - \frac{\mathbf{n}}{r} \times \mathbf{A}(t) = - \frac{\mathbf{n}}{r} \times \int_V \frac{\mathbf{J}(t)}{cr} dV = \int_V \frac{\mathbf{J}(t) \times \mathbf{n}}{cr^2} dV. \qquad (37.15)$$

Canceling the common factor $\exp(-i\omega t)$, we obtain for the amplitude of the potential magnetic intensity

$$\mathbf{B}_{\omega pot} = \int_V \frac{\mathbf{J}_\omega \times \mathbf{n}}{cr^2} dV. \qquad (37.16)$$

This is the magnetic potential of a stationary (quasi-static) system of electric charges, as it can be immediately shown taking rotation from $\mathbf{A} = \int \mathbf{J} dV/cr$.

The radiation electric and magnetic intensities (37.9) and (37.12) can be immediately obtained from formulas (34.35), which we can write in the form

$$\mathbf{E}_{rad} = \mathbf{n} \times (\mathbf{n} \times \dot{\mathbf{A}}/c), \qquad \mathbf{B}_{rad} = - \mathbf{n} \times \dot{\mathbf{A}}/c, \qquad (37.17)$$

in which form they are valid if $\mathbf{A}$ is the advanced magnetic potential not only of a single charge but of a whole system. Indeed, if we put here (36.14), using (37.6) and (37.7), we easily obtain (37.9) and (37.12).

As said above, conventional physics has to consider the last terms on the right sides of equations (37.8) and (37.11) together with the factor exp(ikr). This will give to the potential electric and magnetic intensities a "wave character". A very easy experiment showing that this is not true, i.e., that the potential electromagnetic intensities have no "wave character" is the following one: Take two big coils set aside at a certain distance L and feeded by strong currents with the same high enough frequency, so that $c/\omega < L/2\pi$. Take another small coil closed shortly by an amperemeter in which current will be induced and so it will serve as an indicator of the potential electric field produced by the big coils. If moving the indicator coil between both powerful coils, we shall see that the induced current is the largest when the small coil is near the one or the other coil and gradually decreases, being the less at the middle point. If the potential magnetic field would have a "wave character", the induced current will not decrease gradually at the above motion of the small coil, as both potential fields will interfere and the indicator has to show "nodes" and "anti-nodes" of the produced "standing waves". Nobody nowhere has observed such an effect. This effect, however, can be very easily observed exactly in the above way for the radiation electromagnetic field of two antennas.

Now the big question is to be posed, how can we, by measuring a certain electric intensity E and a certain magnetic intensity **B**, discern which is potential and which is radiation (or which parts in E and **B** have potential and which radiation character). This is a very important question to which official physics cannot give a clear answer.

The distinction which I make is the following: **E** and **B** are radiation elecric and magnetic intensities if and only if they are produced by the same charges, have equal magnitudes, are mutually perpendicular, and the vector **E**×**B** points away from the system producing them. Note that the requirement "produced by the same charges" is very important. So if we have a parallel plates condenser producing the electric intensity **E** and a cylindrical current coil whose axis is perpendicular to **E** producing a magnetic intensity B such that B = E, then the requirement of calling them radiation electromagnetic intensities are fulfilled except the requirement to be produced by the same charges. Thus these electric and magnetic intensities are potential.

The requirement "produced by the same charges" in the above definition can be replaced by the following one: On a unit surface placed perpendicularly to the vector **E**×**B**, a pressure must act equal to the pressure which a gas with mass density $\mu = E^2/4\pi c$ moving with velocity 1 cm/sec exerts on a wall placed perpendicularly to its flow. Thus the radiation electric and magnetic intensities must transfer energy (mass).

I sketched in fig. 17 another experiment which can demonstrate the substantial difference between potential and radiation intensities.

Let us have an oscillating circuit consisting of an induction coil L, a conden-

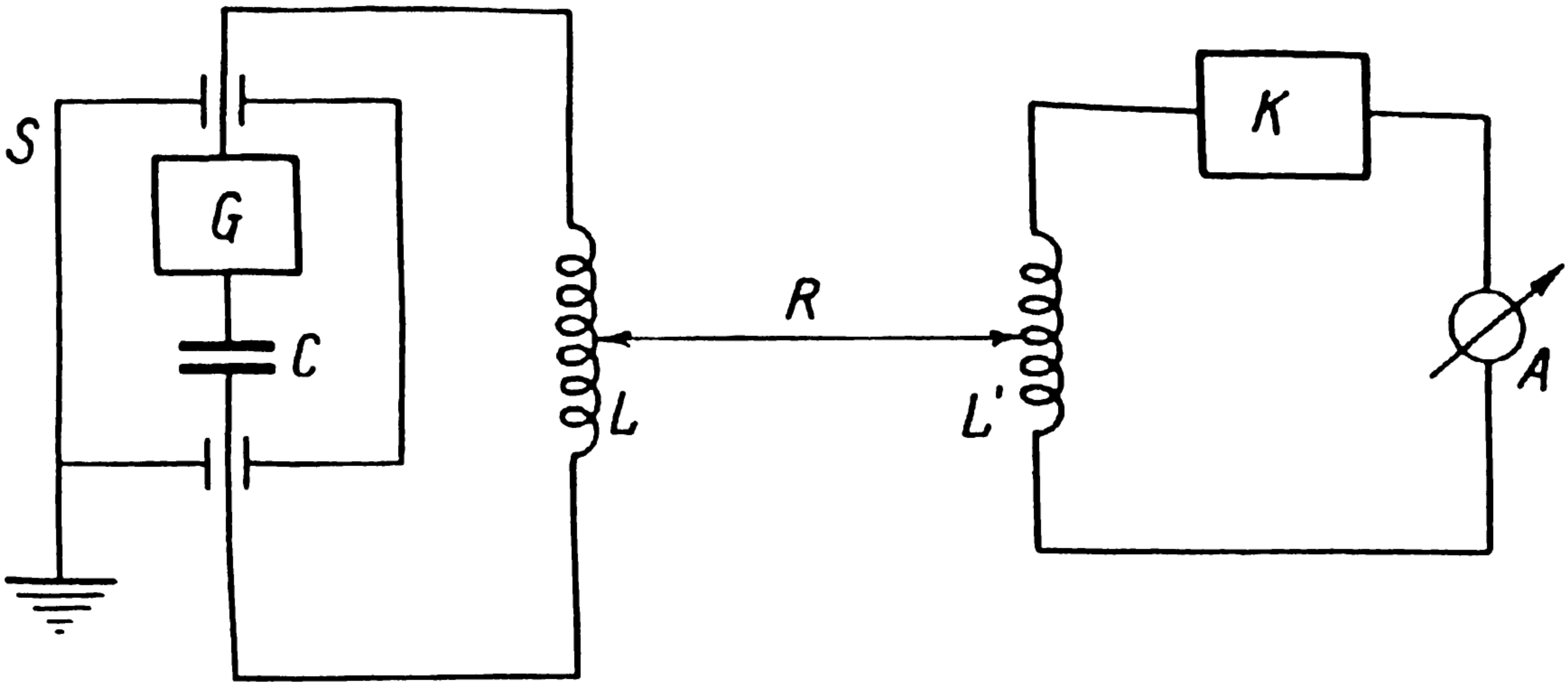

Fig. 17. Experiment demonstrating the momentary propagation of the potential magnetic intensity.

ser C and a generator G which maintains undamped electromagnetic oscillations of the circuit. As it is known, the period of oscillations and the circular frequency are given by the formulas (see Sect. 54.2)

$$T = 2\pi(LC)^{1/2}, \qquad \omega = 2\pi/T = (LC)^{-1/2}. \tag{37.18}$$

Let us suppose that the condenser and the generator are enclosed in a screen-box S, so that this oscillating circuit cannot radiate electromagnetic waves into free space, where only its potential magnetic field will exist.

Let us put another induction coil L' at a distance R from the coil L. If coil L is long enough, we can assume that its potential magnetic intensity will be concentrated in the coil pointing along its axis and having the value $B = (4\pi nI/c)\cos(\omega t)$, where n is the number of the windings on a unit of length and I is the amplitude of the alternating current flowing in the windings (see formula (18.28)). The magnetic potential of L at the space domain where L' is placed is $A = (2\pi nIr^2/cR)\cos(\omega t)$, where r is the radius of the coil L. The magnetic potential A is tangential to a cylinder with raduis R having the same axis as the axis of coil L. According to the first formula (34.1), the electric intensity generated by the alternating current in L at the domain where L' is placed will be also tangential to the mentioned cylinder with radius R and have the magnitude $E = (2\pi nIr^2\omega/cR)\sin(\omega t)$. As in the windings' halves of L' which are nearer to L the induced electric intensity will be bigger than in the halves which are farther, a resultant sinusoidal tension will be induced in L'. This tension, however, is small (if L is infinitely long, it disappears), and it is better to make L' with a radius R <u>encircling</u> L.

Let now suppose that the condition

$$R > cT \tag{37.19}$$

is fulfilled. According to official physics, for the time of one period of the oscillations the field of the magnetic potential propagating from coil L to coil L' cannot reach the latter. But, on the other hand, we know that at the beginning and the end of every half period the whole electromagnetic energy of the circuit is concentrated in the electric field of the condenser C (suppose for simplicity sake that the circuit L-C is without losses which, as a matter of fact, are covered by the energy coming from the generator G). Thus we have to conclude that under the condition (37.19) no electromagnetic energy can be transferred from the circuit L-C to the coil L'.

According to my primitive and childish concepts, the potential electric and magnetic fields do not "propagate" with velocity c but "appear" instantly in whole space. Thus even at the condition (37.19) electromagnetic energy will be transferred from the circuit L-C to the circuit of coil L', and the amperemeter will show the existence of induction current. As the field in the outer space is potential, at open circuit of L' no energy will be absorbed from the potential field and the generator G will cover only the inevitable losses in the circuit L-C. However, if the ciruit of L' will be closed, induced current will flow in it, energy will be absorbed and, because of the back induction of L' in L, the generator must increase its power, otherwise the energy consumed by L' will damp the oscillations in the L-C circuit.

Let us now put the screen box S away and let us begin to make the distance between the condenser's plates bigger and bigger, until the whole circuit will become a straight line with a condenser's plate at any of its ends and the coil L in the middle. If the coil will remain further very long and having the whole magnetic field inside, this system will again have only potential fields in the outer space and both fields (of the condenser and of the coil) will be electric. If, however, we shall begin to diminish the windings of the coil reducing it at the end to a straight wire, in the outer space will exist both the electric and magnetic intensities of the L-C circuit. The parts of them which will be with equal magnitudes, which will be mutually perpendicular and for which the product $E \times B$ will point away from the system will be their radiation electric and magnetic intensities. The coil L' will react both to the potential and radiation electric and magnetic intensities and current generated by their common action will flow in L'.

Here it is to be mentioned that if the predominant part of the energy absorbed by L' will have a radiation character, then the fact whether L' is closed (absorbs energy) or open (does not absorb energy) has no influence on the generator G which covers only the inevitable losses in the circuit and the energy radiated in the form of electromagnetic waves (photons).

All these experiments are enough simple for execution and their explanation is also extremely simple. Nevertheless official physics defends the wrong concept that also the potential electric and magnetic intensities, and even the electric and magnetic potentials, "propagate" with the velocity of light.

At the end of this section I should like to emphasize once more that the potential electric and magnetic intensities are determined by the values of the charge and current densities at the different elementary volumes of the system, while the radiation electric and magnetic intensities are determined by the rate of change of these densities.

38. DIPOLE RADIATION

In zero approximation at large distances from the generating system the magnetic potential can be expressed by the dipole moment of the system according to formula (26.14). Substituting this expression for the advanced magnetic potential into the general formula (37.17) for the radiated electric and magnetic intensities, we obtain

$$E_{rad} = \frac{1}{cr}\, n\times(n\times\ddot{d}), \qquad B_{rad} = \frac{1}{c^2 r}\, \ddot{d}\times\ddot{n}. \tag{38.1}$$

The radiation described by the formulas (38.1) is called DIPOLE RADIATION because the electric and magnetic radiation intensities depend only on the dipole moment of the system (on its second time derivative).

As already said, the radiated electromagnetic waves (photons) are carrying away a definite amount of energy from the radiating system. The intensity of the radiated energy flux is given by formula (34.39). Taking into account the relations (see formulas (34.35)) $B_{rad} = n\times E_{rad}$, $E_{rad}.n = 0$, $E_{rad} = B_{rad}$, we can write

$$I = \frac{c}{4\pi} E_{rad}\times B_{rad} = \frac{c}{4\pi} E_{rad}\times(n\times E_{rad}) = \frac{c}{4\pi} E^2_{rad} n = \frac{c}{4\pi} B^2_{rad} n. \tag{38.2}$$

Taking into account our third axiom, we have to understand the above equation always in the following form

$$I = \frac{c}{T}\int_{-T/2}^{T/2} (E^2_{rad}/4\pi)dt = \frac{c}{T}\int_{-T/2}^{T/2} (B^2_{rad}/4\pi)dt, \tag{38.3}$$

where T is the period of the electromagnetic wave (the period of the photon). Indeed, according to the third axiom, only when time equal to the period of a particle has elapsed can we affirm that the particle has crossed a given surface. For times shorter than the period we cannot say on which side of the surface is the particle.

It is more convenient to express **I** by $\mathbf{B}_{rad}$ (see the right-hand expression in (38.2)) as $\mathbf{B}_{rad}$ can be expressed by $\ddot{\mathbf{d}}$ more simply than $\mathbf{E}_{rad}$ (see (38.1)).

The energy flux of radiation dP in a unit of time into the element of a solid angle dΩ is defined as the amount of energy passing in a unit of time through the element $dS = r^2 d\Omega$ of the spherical surface with center at the frame's origin and radius r (see fig. 16). This quantity is clearly equal to the intensity of the energy flux density I multiplied by dS, so that using (38.1) we obtain

$$dP = IdS = (c/4\pi)B^2 r^2 d\Omega = (1/4\pi c^3)(\mathbf{n}\times\ddot{\mathbf{d}})^2 d\Omega. \tag{38.4}$$

The whole energy flux can be obtained if we integrate (38.4) over a sphere containing the radiating system at its center. Let us introduce spherical frame of reference with polar axis along the vector $\ddot{\mathbf{d}}$. Let the zenith angle and the azimuth angle of the unit vector **n** be θ and ϕ; θ is consequently the angle between $\ddot{\mathbf{d}}$ and **n**. As $d\Omega = \sin\theta \, d\theta \, d\phi$,

$$P = \int_{4\pi} \frac{(\mathbf{n}\times\ddot{\mathbf{d}})^2}{4\pi c^3} d\Omega = \int_0^{\pi}\int_0^{2\pi} \frac{\ddot{d}^2}{4\pi c^3} \sin^3\theta \, d\theta \, d\phi = \frac{2}{3c^3} \ddot{d}^2. \qquad (38.5)$$

If we have just one charge moving in an external field, we shall have, keeping in mind (31.6), $\ddot{\mathbf{d}} = q\ddot{\mathbf{r}} = q\mathbf{u}$, so that the total energy radiated in a unit of time by this charge will be

$$P = \frac{2q^2}{3c^2} u^2. \qquad (38.6)$$

We note that a system of particles, for which the ratio of charge to mass is the same, cannot radiate (by dipole radiation). Indeed, for such a system

$$\mathbf{d} = \sum_{i=1}^{n} (q_i/m_i) m_i \mathbf{r}_i = \text{Const} \sum_{i=1}^{n} m_i \mathbf{r}_i = \text{Const}\, \mathbf{R} \sum_{i=1}^{n} m_i, \qquad (38.7)$$

where Const is the charge-to-mass ratio common for all charges and **R** is the radius vector of the center of mass of the system. As the center of mass moves uniformly, its acceleration is zero and consequently the second time derivative of **d** is zero, too.

If the particle performs such a motion that its dipole moment is a simple periodic function of time with a period $T = 2\pi/\omega$, we shall have

$$\mathbf{d}(t) = \mathbf{d}_\omega e^{-i\omega t}, \qquad (38.8)$$

where $\mathbf{d}_\omega$ is the complex amplitude of the dipole moment (which, at a suitable choice of the initial moment, can be taken real and equal to the maximum value of the dipole moment - see Sect. 35).

Hence, substituting (38.8) into (38.5), we obtain for the total energy flux

$$P = \frac{2}{3c^3} |\ddot{\mathbf{d}}(t)|^2 = \frac{2}{3c^3} \omega^4 |\mathbf{d}_\omega|^2. \qquad (38.9)$$

39. RADIATION REACTION

As formulas (34.47) show, the radiation reaction electric and magnetic intensities are as follows

$$\mathbf{E}_{rea} = -(2q/3c^3)\mathbf{w}, \qquad \mathbf{B}_{rea} = 0. \qquad (39.1)$$

Let us calculate the change of the energy of a system of n charges due only to the action of the electric intensities of radiation reaction $\mathbf{E}_{rea_i}$ of the various charges. On each charge of the system the "kinetic" force